A model-theoretic approach to characterizing randomness notions

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Warmup: Notions of randomness from $G(\omega, p)$ and for $\mathscr{R}$

$\mathbf{G}(\omega, \mathbf{p})$: Erdős–Rényi random variable with vertex set ω and edges with independent probabilities p.

$\mathscr{R}$: Rado graph, i.e., the "random graph", obtained (up to isomorphism) by $G(\omega, p)$ with probability 1 (for $0 < p < 1$).

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Q: Consider the image of $G(\omega, p)$ for, say, a ML random point in the basic space. In what sense does this constitute an algorithmically random presentation of $\mathscr{R}$?

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Q: There are many other probabilistic constructions of $\mathscr{R}$. Do these yield different measure-one sets of points in the basic space (upon taking the preimage of the isomorphism class of $\mathscr{R}$)?

Q: Taking these sets to constitute notions of randomness, do we recover any well-known classes? Do we obtain any new ones?

Q: What if we consider other structures or classes of structures (e.g., models of a given theory) and their probabilistic constructions?

Talk Outline

Two points of view:

1. Probabilistic constructions provide new examples of “almost everywhere” theorems, using which we can **recharacterize old notions of randomness** or **discover new ones**.

2. We can use symmetric computable probabilistic constructions of unordered countable structures to formulate notions of **algorithmically random such structures**.

Background on computability of graphons

Characterizing μ-Martin–Löf (for continuous μ) and Kurtz randomness

Open questions:

Recovering familiar randomness notions

The poset of such randomness notions

Algorithmically random countable structures

Effective dimension

Powerful consequences of statements about randomness