A Heuristic Method for Searching Optimum Path in Contamination Matrices

Felipe Maldonado [1] *Luis B. Morales* [2] *Augusto Ciurlizza* [1]

[1]Escuela Superior de Ingeniería Textil
Instituto Politécnico Nacional
Insurgentes Sur 3493 Edif. 27-503, D.F., 14020, México
Becarios de COFAA

[2]IIMAS, Universidad Nacional Autónoma de México

Abstract
Contaminating processes are present in various manufacturing industries, whose effects are attempted to minimize. A typical case corresponds to dyeing of fabrics in the textile industry. Modeling of the process leads to matrices which, though corresponding to the Asymmetrical Traveling Salesman Problem, have characteristics that facilitate the search process, allowing discovery of the optimum path with a short range search. In this paper, we present a model that allows the use of optimization techniques to minimize the dyestuff pollution effect, and heuristic considerations facilitating the industrial task of determining the contamination matrix.

Keywords: Contamination, Traveling Salesman Problem, Dyeing, Color.

1 Introduction

The traveling salesman problem (TSP) is one of the mathematical problems that has received a high degree of attention because it has multiple scientific and engineering applications, being itself quite simple. Given a set of cities $\{1, \ldots, n\}$ and given the cost of traveling between every pair of them in an $n \times n$ matrix (c_{ij}) form, the traveling man dilemma consists in finding a minimum-cost tour that visits each city one time only, and returns to the start point. But, in spite of its "simplicity", all intents made to date have failed to find an efficient algorithm to solve it. We consider an algorithm efficient if its execution time is bounded by a polynomial function written in terms of some reasonable measure of the problem size. Problems such as TSP are called NP-complete.

A very important case, also with many applications [6, 12], is when the costs matrix is asymmetrical; that is $c_{ij} \neq c_{ji}$. This one is called the asymmetrical traveling salesman problem (ATSP).

There is a tendency oriented toward searching for approximate and heuristic algorithms, which, though they do not always solve the problem, allow near solutions that are often acceptable from the point of view of concrete applications. Our task is also directed to this.

The original problem we attacked consisted in finding optimum color sequence in dyeing fabric. We developed a model that allows to quantify differences among groups of colors, making the use of optimization methods possible [11]. The model leads to a similar problem to that of ATSP, but without returning to the first city visited. We solved the problem of finding optimum color sequence for programming fabric dyeing in short color series [11] in an exact manner, using the sequence cost indicated by experts as an initial bound.

The sequence proposed by experts provided a satisfactory initial bound; however, in spite of the "kindness" of the initial sequence, the exhaustive search faces two problems. One is due to the intrinsic ATSP characteristics, in which machine-time increases exponentially with the number of colors. The other, practical as well, is because the quantity of cost value determinations, carried out on a color-gauging equipment, increases with the square of the number of colors.

The determinations carried out allowed us to prove that if the initial sequence is chosen well, it is not necessary to conduct an exhaustive search. We developed a heuristic method, which we have compared successfully to the exact method. The method implemented is a short-range method which has the advantage of resolving the problem in a shorter time than the exhaustive, and may be applied to longer color series, including those having multiple restrictions [7, 8, 9]. Our method has the additional advantage of being able to be utilized without counting on all the costs, knowing only those corresponding to a band parallel to the main diagonal, which noticeably diminishes the number of determinations to be carried out on the colorimeter.

On the other hand, the determinations carried out allow us to propose criteria on what the depth of the search should be, in order to find the global optimum with a higher degree of certainty.

In Section 2, we briefly presented the model which makes quantifying differences among color sets possible [11] and basic considerations about color space. In Section 3, we developed heuristic criteria allowing us to reach, to a high percentage, global optimum with a short-range search and give the obtained results.

2 Problem Formulation

Industrial dyers receive orders from clients to dye fabrics in various colors. Technical personnel make out a list of colors to be dyed sequentially. Sequence selection is carried out based on the "lightness" and "cleanliness" of colors and, above all, their own experience. All fabrics are dyed in one machine only, which is emptied, but not washed after each process. After dyeing each fabric, the majority of dyestuff utilized is impregnated in it, but a small percentage remains in the dyeing equipment. The amount consumed in the process depends on the nature of the dyestuff and operational parameters, such as time and temperature. The non-consumed fraction constitutes a contaminating factor affecting the next dyeing process. That is to say, the color of the next fabric to be dyed can differ from the desired color due to the presence of residue of previously used dyestuff. The degree to which residues affect the following processes depends on the color of the residual material and the new color to be dyed; which means that, from the point of view of consequences it has on the dyeing, each pair of colors constitutes an asymmetrical dyad.

We present here some elements about colors and their determination. The color complexity proceeds from its psychophysical characteristics. A numerical form must be given to the human sensation of color, that is known as the perceptual response of the observer, and this must be related to the spectrophotometric values, which depends on the energy of the light source and the refraction of the sample.

The first quantitative form of human sensation of color dates back to 1931, and was made by the Commission Internationale de l'Éclairage (CIE). After many transformations we arrived at a system proposed by Hunter, which is currently widely accepted, and called the *opposing color system*. Its importance resides in the fact that it permits the use of color difference equations. In this study we used the opposing color system, which is a rectangular cartesian coordinate system. It consists of a vertical L-axis, of "light-darkness", and two horizontal a- and b-axes, which correspond to "red-green" and "blue-yellow" . With each point in color space there is associated an ordered triple of numbers, called color coordinates. The first number in the triple indicates the location of the point in the direction of the L-axis, the second one indicates its location in the direction of the a-axis, and the third number indicates the location of the point in the direction of the b-axis. The accepted term identifying the system is the CIELab [1, 2, 4].

We now present a mathematical model for the fabric dyeing process. Given a set of colors $\{1, \ldots, n\}$ and n fabrics, which must be dyed in these colors, the problem is to find the processing sequence which produces the minimum differences between desired colors and those obtained. Formally, when dyeing with color j after dyeing with color i a cost c_{ij} is produced (each c_{ij} value represents the cost of dyeing color j after color i). This cost c_{ij} is measured by the euclidean distance between the coordinates CIELab of both colors; that is,

$$c_{ij} = \sqrt{(L - L')^2 + (a - a')^2 + (b - b')^2},$$

where (L, a, b) and (L', a', b') are the coordinates of the desired and obtained colors, respectively. The matrix (c_{ij}) is called *matrix of contamination.*

Clearly, each sequence of the n colors to be dyed corresponds to a permutation π of the set $\{1, \ldots, n\}$. Hence, a permutation $\pi = (\pi(1), \ldots, \pi(n))$ defines a sequence of the colors to be dyed; that is to say, $\pi(1)$ will be the first color to be dyed, $\pi(2)$ will be the second color to be dyed and so on, successively, up to $\pi(n)$ color to be dyed. If π^{-1} is the inverse of the permutation π, then $\pi^{-1}(i)$ is the position of color i in the sequence of colors to be dyed. Then, the problem of finding an optimal color sequence can be formulated as the optimization problem

$$\text{minimize } _{\pi} z(\pi) \sum_{i=1}^{n-1} c_{\pi(i)\pi(i+1))} \tag{1}$$

Thus, the minimization of the pollution effect in the dyeing of fabrics leads to the asymmetrical salesman traveling problem, but the tour should not be closed; that is, the first fabric dyed should not be re-dyed in the established sequence.

3 Methodology and Results

Once the contamination matrix (c_{ij}) has been determined, we confront the problem of optimizing the relation (1). Though there are exact methods that can now be run on a PC, (see for example [3, 10] to solve our ATSP for any normal-sized set of colors in the industry (< 50 colors), a practical sequence problem at an industrial level subsists. The determination of each c_{ij} value is effected by a simulation process, for a given contamination level, on a colorimeter. Therefore, for a set of 50 colors, it would be necessary to determine 2500 c_{ij} values, which would be considerably time consuming.

However, heuristic considerations allow determining optimal (or very near to optimal) sequence utilizing only a small part of the total of the contamination matrix (c_{ij}).

Sequential dyeing of fabrics in different colors has characteristics which allow identification by empirical criteria, that experts use on an everyday basis since a century ago, such as color "clarity" and "cleanliness", sequence whose cost is near the global optimum. Consequently, they are "good" initial solutions for an exhaustive searching process. These criteria allow obtaining comparatively better initial solutions than those found through other method.

We present, in Table 1, a typical example of a 37-color set with its CIELab coordinates ordered from 1 to 37, according to expert criteria. Figure 1 gives the contamination matrix for this set; each c_{ij} value represents the cost of dyeing color j after color i. The cost of experts sequence is 24.31, and it is lesser than other cost sequences such as random sequences, ordered by a greedy or ordering colors from light to dark in the L-axis (CIELab space). Table 2 gives the distinct ordering costs and optimum sequence cost of this 37 colors set.

Number	L	a	b	Number	L	a	b
1	96.56	-23.07	68.93	20	30.38	-6.84	-1.03
2	91.86	13.70	32.20	21	35.02	17.31	-34.74
3	87.02	-15.10	78.02	22	48.98	-36.38	10.19
4	84.02	-5.10	70.02	23	43.03	-9.76	10.04
5	81.70	4.90	84.71	24	20.03	-8.04	12.47
6	72.69	5.10	31.05	25	54.21	46.20	38.64
7	60.03	26.03	-7.85	26	46.03	-23.61	7.08
8	38.69	52.73	26.16	27	40.20	36.31	30.65
9	42.01	40.13	15.55	28	20.67	20.24	-25.63
10	57.61	35.27	63.59	29	20.33	-3.04	2.47
11	47.16	44.07	-12.46	30	41.05	-4.76	2.78
12	58.02	26.12	60.80	31	20.45	12.61	-16.86
13	52.11	39.68	54.06	32	43.14	-1.44	-24.32
14	52.68	34.66	28.00	33	39.47	-3.61	-2.49
15	25.18	-8.04	-1.32	34	32.31	-9.10	-1.07
16	46.79	22.11	16.32	35	46.60	-25.36	6.51
17	41.17	1.25	-40.97	36	40.24	-3.40	-14.12
18	25.36	10.18	-28.02	37	20.26	-9.64	-1.32
19	27.16	13.89	-30.04				

Table 1. The L, a, b values are the coordinates of each color in the CIELab space

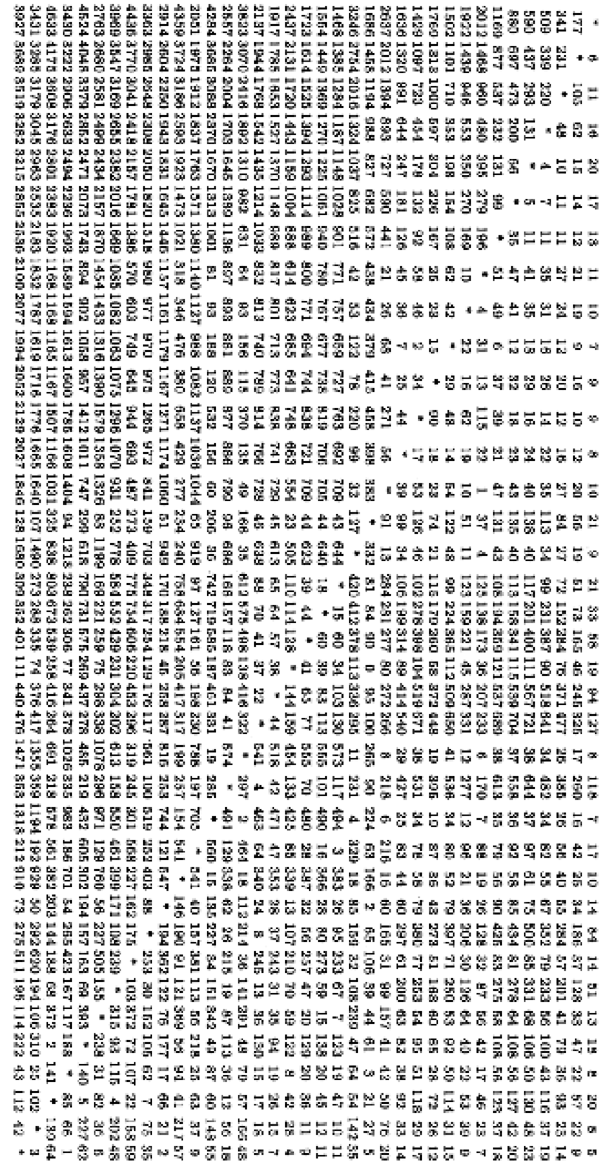

Fig. 1: Matrix of contamination ($100 \times (c_{ij})$). Each c_{ij} value represents the cost of dyeing color j after clor i.

Fifty determinations with matrices ranging from 10 to 37 colors were made, with the same order of costs obtained in every case.

$$global\ optimum < experts < greedy < L-axis < random$$

Proposal	Cost	Relative Cost
Optimum	11.37	1
Experts	24.31	2.14
Greedy	27.29	2.40
L-axis	71.18	6.26
Random	225.97	19.87

Table 2. Cost of different initial successions compared to the global optimum

It is also observed that if colors are ordered adequately from the beginning, resulting c_{ij} values utilized in the determined optimum sequence for some exact method are found within a band parallel to the main diagonal, and the band width is narrower if initial order is that recommended by experts.

These results are transcendental because it is implied that the way for optimum color sequence is found within a narrow band if colors are ordered adequately from the beginning. That is to say, the c_{ij} values involved will be only those adjacent to the main diagonal, and therefore, it is not necessary to know all c_{ij} values to find optimum sequence.

Determination for different color group sizes was carried out with the objective of determining the needed band-width. Fifty groups were analyzed, ranging from 10 to 37 colors, and in all cases, band-width was determined where the c_{ij} values finally involved in the optimum sequence were found. The results obtained vary from the halfway point of the matrix ($.6n$) for 10 colors to one third of the matrix order for larger groups used ($.3n$). The results are shown in Table 3.

Quantities of colors	Band–width
10	6
15	8
20	10
25	10
30	10
37	11

Table 3. Average value of band-width in which (c_i) value are found that participate in the optimum succession for different quantities of colors.

Using this heuristic criteria, we made an iterative search with Q colors ($Q =$ band width $< n$). We begin with the first Q colors in the original sequence. Of the optimum sequence obtained, the first color in the series remained fixed, and a new group of Q colors was completed with the following color in the original sequence. The procedure led to optimum sequences, which is to say, with the same cost as that obtained by the exact method, in all cases. In Table 4, we present the results obtained, in a set of 37 colors, with different depths.

Depth of the search "Q"	Cost
13	11.37
12	11.37
11	11.37
10	12.58
9	18.71

Table 4. Values for optimum succession cost (for the 37-color group) depending upon the depth of the search.

The results obtained for a group of 37 colors are in Table 5. In all cases the optimum value was first obtained with an exact method and this value was used as a comparing element with values obtained with the short range method.

Optimum order	Depth of search	Cost
1,2,3,4,5,6,7,12,10,13,11,9,8,14,16,23,25,27,30,35, 33,31,2, 9,24,22,21,19,18,17,26,32,36,28,15,20,34,37	11	11.37

Table 5. Optimum succession found through the short-range method.

An "a priori" awareness exists in costs matrices corresponding to fabric-dyeing of characteristics that allow proposing solutions close to the global optimum. Then, it is reasonable to think that ATSP cost matrices do exist, with a different origin than that of fabric dyeing, but with similar characteristics; that is to say, with intrinsic characteristics permitting the selection of a path close to optimum arrangement; and this suggest the possibility of resolving those ATSP instance in polynomial time. This will be object of a future paper.

4 Conclusions

(1) The proposed model relating difference between dyed colors with the color sequence used and assigning a value in terms of the difference of its coordinates allows confronting use of optimization methods.

(2) Upon achieving an optimum color sequence, in terms of minimizing differences between colors obtained and colors desired, end products quality increases and shading reprocessing decreases, with has favourable and direct repercussions on costs.

(3) Developing a short-range method allows resolving extended color series with a limited number of costs c_{ij}, making the method more accessible to the industrial sector.

Acknowledgment

This work was partially supported by the Project CONACYT 2133-PA.

References

[1] Color Measurement, Principles and the Textile Industry, (Research Triangle Park, NC: AATCC, 1991).

[2] BEST, R., Colouring by numbers, *Textile Horizons* 8, (Aug 1986).

[3] CARPANETO, G., DELL'AMICO, M., and TOTH, P., "Exact solution of large-scale, asymmetric traveling salesman problems" *ACM Trasns. Math. Softw.* 21 (1995), 394-409.

[4] CELIKIZ, G. and KUEHNI, R.G. *Color technology in the textile industry*, USA., 1983.

[5] LAWLER, E.L., LENSTRAL, J.K., RINNOOY KAN, A.HG. and SHMOYS, D.B., *The traveling salesman problem: a guided tour of combinatorial optimization*, Wiley Chichester, 1985.

[6] MADSEN, O.B.G., "An application of the traveling-salesman routines to solve pattern-allocation problems in the glass industry", *J. Oper. Res. Soc.* 39 (1988), 249-256.

[7] MALDONADO, F., "Análisis numérico para la formulación de colores", In: IV Congreso Nacional de Ingeniería Textil, Buenos Aires, Argentina, 1996.

[8] MALDONADO, F., "Aplicación industrial del agente viajero asimétrico con restricciones múltiples", In: VIII CLAIO, Vol. 2, pp. 492-497, Río de Janeiro, Brasil, 1996.

[9] MALDONADO, F., CIURLIZZA, A. and MORALES, L.B., "Aplicación industrial del agente viajero asimétrico en caminos no-simplemente conexos", In: 1er Encuentro Latino Ibero Americano de Optimización, pp. 62-66, Universidad de Concepción, Chile, 1997.

[10] MILLER, D.L., and PEKNY, J.F., "Exact solution of large asymmetric traveling salesman problem", *Science* 251 (1991), 754-761.

[11] MORALES, L., MALDONADO, F., RADILLO, R. and CIURLIZZA, A., "Optimisation of the color sequence in the process of fabric dyeing", *Journal of the Society of Dyers and Colourists*, 112 (1996), 361-363.

[12] PLANTE, R.D., LOWE, T.J. and CHANDRASEKARAN, R., "The product matrix traveling salesman problem: an application and solution heuristic", *Opns. Res.*, 35 (1987), 772-783.