A Framework for Enhanced Indexing in the Brazilian Stock Market

Antonio Marcos Duarte Júnior

Diretor, Gerenciamento de Riscos Corporativos
Unibanco S.A.
Rua Pio Correia, 110 / 105
22461-240 Rio de Janeiro, RJ, Brazil
Phone: 55-21-2860224
antonio.duarte@unibanco.com.br

Abstract
Enhanced indexing is a hybrid fund management style that presents elements of both the active and the passive investment styles. This work presents the basic principles of enhanced indexing fund management, and proposes a methodology to generate the set of its efficient portfolios. Important operational details for practitioners are addressed, such as choosing an appropriate market risk measure, and the mathematical formulation of the optimization models used to generate the set of efficient portfolios. The practical use of enhanced indexing fund management is illustrated in the Brazilian stock market.

Resumo
Indexação com vício é um estilo híbrido de administração de fundos de investimentos que apresenta elementos tanto da administração ativa de recursos, quanto da administração passiva de recursos. Este trabalho apresenta os princípios básicos da indexação com vício, além de propor uma metodologia para obtenção do conjunto de carteiras eficientes. Aspectos operacionais importantes na prática são abordados, como a escolha de uma medida de risco apropriada, além da formulação matemática dos modelos de otimização utilizados. O uso prático de indexação com vício para administração de recursos é ilustrado no caso do mercado acionário brasileiro.

Keywords: Asset allocation; portfolio management; risk management.

---

1 Introduction

Fund management styles have been traditionally classified as active or passive ([4]). Active management seeks to attain above average risk-adjusted performance. Passive management seeks to attain the market average risk-adjusted performance. Essential to active management are expectations about the future performance of assets.

Passive management requires no expectational input.

Classifying fund managers according to these two styles is becoming increasingly difficult ([5]). For example, although indexing portfolios is classified as a passive investment style, and stock picking to maximize long term returns is classified as an active investment style, enhanced indexing does not fit in neither style. It is for this reason that enhanced indexing is most often considered a hybrid investment style that presents elements of both the passive and the active investment styles.

Enhanced indexing is a investment style particularly suitable for fund managers who measure their performance with respect to a benchmark on a continuous basis. Enhanced indexing allows these persons to seek an above average risk-adjusted performance when desired, returning to the market average risk-adjusted performance when appropriate, while keeping a strict risk control with respect to deviations from the benchmark.

This work proposes a methodology to generate the set of efficient portfolios for enhanced indexing fund management. Efficient portfolios are those that minimize the expected risk for a given level of expected return, and maximize the expected return for a fixed level of expected risk ([9]). Three important characteristics of this methodology are:

1) It covers both passive and active investment styles. For example, passive investment strategies, such as indexing portfolios, are obtained as a special case. Active investment strategies, such as stock picking to maximize long term returns, are also obtained as a special case. The transition from a passive investment style to an active investment style is reduced to the problem of choosing among different portfolios in the set of efficient portfolios.

2) Market risk is measured with respect to a index selected as the benchmark, and normalized to vary between zero and one. Interpreting the market risk measure in this case is similar to interpreting the R-Squared of linear regression models ([13]): the closer the market risk is to one, the more the benchmark adopted explains the market exposure of the portfolio; the closer the market risk is to zero, the less the benchmark adopted explains the market exposure of the portfolio.

3) The methodology used to generate the set of efficient portfolios is based on recent proposals in the portfolio optimization literature. A scenario-based optimization model is used to obtain the set of efficient portfolios. It consolidates in a single methodology scenario-based optimization models for active fund management ([8]) and passive fund management ([3]). As a result, the set of efficient portfolios is obtained at no extra mathematical modeling complexity or computational costs when compared to fast implementations of portfolio optimization models ([10,11]).

This work is divided as follows. The next section addresses the problem of finding an adequate market risk measure for enhanced indexing fund management. Following that, the mathematical formulation of the optimization problems used to generate the set of efficient portfolios are presented. A numerical example illustrates the practical use of these optimization problems in the case of the Índice Geral da Bolsa de Valores de São Paulo (IBOVESPA). Final comments are provided in the conclusion.

2 Market Risk Measurement

Choosing a market risk measure is crucial for the effective market risk management of any investment fund. In order to satisfy investment objectives, market risk must be measured in accordance with the policy guidelines of funds. Four examples are:

1) A portfolio manager responsible for a global bond index fund whose objective is to track the Salomon Brothers World Government Bond Index (SBWGBI). In this case, market risk should be measured using the volatility of the tracking errors with respect to the SBWGBI ([6]). A small volatility of the tracking errors is related to a good tracking of the SBWGBI.

2) The trustees of an endowment portfolio whose objective is to meet future liabilities, while keeping long-term purchasing power. In this case, market risk should be measured using the downside risk with respect to the trustees' minimum acceptable return ([7]). A small downside risk is related to meeting the trustees' investment objectives.

3) An active equity manager whose objective is to beat the New York Stock Exchange Composite Index (NYSECI). In this case, market risk should be measured using the downside risk with respect to the returns of the NYSECI ([2]). A small downside risk is related to beating the NYSECI.

4) A portfolio manager responsible for a global hedge fund whose objective is "to make money in any market environment." In this case, market risk should be measured using the standard deviation of the possible returns. A small standard deviation is related to a small chance of experiencing huge losses.

Before presenting the market risk measurement framework used to monitor the exposure of efficient portfolios, it is necessary to define some notation. Although the presentation in this work concentrates in equity markets, it can be easily adapted to other markets.

We assume that $n + 1$ assets are used to obtain the set of efficient portfolios: a cash-equivalent instrument (such as certificates of deposit) and n stocks. If the amount available for investing is one monetary unit, the equilibrium equation is

$$Xc + \sum_{i=1}^{n} X_i = 1, \tag{1}$$

where Xc is the final amount invested in cash-equivalent instruments, and

$X_1, X_2, ..., X_n$ is the amount invested in each of the n stocks used. In order to keep the presentation simple, the model used in this work assumes no leverage, no borrowing, no transaction costs, no taxes, and no short-sales. These modeling aspects can be incorporated in the model using the suggestions given in [12].

We assume that m scenarios are available. These can incorporate investors views as proposed in [1] and [8], or only historical information. Let R_C denote the expected return of the cash-equivalent instrument for the investment horizon considered. Let R_{ij} denote the expected return of the i^{th} stock according to the j^{th} scenario for the investment horizon considered. If P_j denotes the expected return of the efficient portfolio according to the j^{th} scenario, it is given by

$$P_j = R_C Xc + \sum_{i=1}^{n} R_{ij} X_i \quad \forall\, j = 1, 2, ..., m \tag{2}$$

Generating the set of efficient portfolios requires selecting a stock index to be the benchmark. Let I_j denote the expected return of this benchmark according to the j^{th} scenario. The expected tracking error E_j for the j^{th}scenario satisfies

$$E_j = P_j - I_j \quad \forall\, j = 1, 2, ..., m. \tag{3}$$

If $E_j < 0$, the benchmark should outperform the efficient portfolio according to the j^{th} scenario; if $E_j > 0$, the efficient portfolio should outperform the benchmark according to the j^{th} scenario. Optimally indexing a portfolio can be achieved by minimizing an estimate for the volatility of the tracking errors. To that end, it is sufficient to minimize $\sum_{j=1}^{m} E_j^2$. Beating the benchmark can be achieved by maximizing the expected returns for the scenarios used. To that end, it is sufficient to maximize $\sum_{j=1}^{m} P_j$. These two problems will be considered in details later.

Market risk is measured using

$$R^2 = 1 - \frac{\sum_{j=1}^{m} (I_j - P_j)^2}{\sum_{j=1}^{m} (I_j - R_C)^2} = 1 - \frac{\sum_{j=1}^{m} E_j^2}{\sum_{j=1}^{m} (I_j - R_C)^2}. \tag{4}$$

It can be verified ([14]) that $0 \leq R^2 \leq 1$. Moreover, since the market risk measure in (4) is fairly similar to the R-Squared of multiple linear regression models ([13]), it can be easily interpreted: the closer the R^2 is to one, the more the benchmark explains the market exposure of the portfolio. Another way to put it is to say that the closer the R^2 is to one, the more "linked" (or perhaps, "indexed") the portfolio is to the benchmark. A justification for this interpretation requires a comparison between the two summations in (4):

1) The numerator $(\sum_{j=1}^{m} E_j^2)$ measures the tracking error of an efficient portfolio using stocks and a cash-equivalent instrument.

2) The denominator ($\sum_{j=1}^{m} (I_j - R_C)^2$) measures the tracking error of a portfolio totally invested in the cash-equivalent instrument. That is, the denominator measures by how much the cash-equivalent instrument fails to track the benchmark.

Allowing the use of stocks to track the benchmark results in a smaller volatility of the tracking errors. The more linked the final portfolio is to the benchmark, the smaller the numerator ($\sum_{j=1}^{m} E_j^2$) is and, consequently, the closer the R^2 is to one.

Now that market risk measurement has been presented, it is possible to describe the methodology to obtain the set of efficient portfolios.

3 A Methodology to Obtain the Set of Efficient Portfolios

The shape of the set of efficient portfolios is depicted in Exhibit 1. The two extreme cases in this set are:

1) The "indexed portfolio" presents the maximum R^2. It is the portfolio that best tracks the benchmark.

2) The "maximum return portfolio" presents the minimum R^2. It is the portfolio that maximizes the expected return for the scenarios used.

The two initial steps when constructing the set of efficient portfolios is to obtain these two extreme cases. This can be achieved by solving two optimization problems.

The problem of optimally indexing a portfolio can be modeled as

$$
\begin{aligned}
&Maximize\ R^2 \\
&subject\ to: R^2 = 1 - \frac{\sum_{j=1}^{m} E_j^2}{\sum_{j=1}^{m} (I_j - R_C)^2} \\
&\qquad E_j = P_j - I_j \quad \forall\ j = 1, 2, ..., m \\
&\qquad P_j = R_C X c + \sum_{i=1}^{n} R_{ij} X_i \quad \forall\ j = 1, 2, ..., m \\
&\qquad Xc + \sum_{i=1}^{n} X_i = 1 \\
&\qquad X_C, X_1, X_2, ..., X_n \geq 0 \\
&\qquad R, E_1, E_2, ..., E_m, P_1, P_2, ..., P_m \in \Re.
\end{aligned} \tag{5}
$$

The output of this optimization model is the "indexed portfolio" in Exhibit 1.

The problem of maximizing the expected return can be modeled as

$$
\begin{aligned}
&Maximize\ \sum_{j=1}^{m} P_j \\
&subject\ to: R^2 = 1 - \frac{\sum_{j=1}^{m} E_j^2}{\sum_{j=1}^{m} (I_j - R_C)^2} \\
&\qquad E_j = P_j - I_j \quad \forall\, j = 1, 2, ..., m \\
&\qquad P_j = R_C Xc + \sum_{i=1}^{n} R_{ij} X_i \quad \forall\, j = 1, 2, ..., m \\
&\qquad Xc + \sum_{i=1}^{n} X_i = 1 \\
&\qquad X_C, X_1, X_2, ..., X_n \geq 0 \\
&\qquad R, E_1, E_2, ..., E_m, P_1, P_2, ..., P_m \in \Re.
\end{aligned}
\tag{6}
$$

The output of this optimization model is the "maximum return portfolio" in Exhibit 1.

Once the two extreme cases have been obtained, the set of efficient portfolios can be generated for intermediate values of R^2. This is achieved by choosing a value for R^2 between the risk of the two extreme cases, defining the set of all portfolios that have R^2 equal to the level of risk desired, and picking the portfolio in this set with the maximum expected return.

The problem of maximizing the expected return forcing the risk to be equal to ρ can be modeled as

$$
\begin{aligned}
&Maximize\ \sum_{j=1}^{m} P_j \\
&subject\ to: R^2 = \rho \\
&\qquad R^2 = 1 - \frac{\sum_{j=1}^{m} E_j^2}{\sum_{j=1}^{m} (I_j - R_C)^2} \\
&\qquad E_j = P_j - I_j \quad \forall\, j = 1, 2, ..., m \\
&\qquad P_j = R_C Xc + \sum_{i=1}^{n} R_{ij} X_i \quad \forall\, j = 1, 2, ..., m \\
&\qquad Xc + \sum_{i=1}^{n} X_i = 1 \\
&\qquad X_C, X_1, X_2, ..., X_n \geq 0 \\
&\qquad R, E_1, E_2, ..., E_m, P_1, P_2, ..., P_m \in \Re.
\end{aligned}
\tag{7}
$$

The output of this optimization model is an efficient portfolio in Exhibit 1 with R^2 equal to ρ (where ρ varies between the R^2 of the "indexed portfolio" and that of the "maximum return portfolio"). One optimization problem needs to be solved to generate each point on the set of efficient portfolios between the two extreme cases.

A practical example of a set of efficient portfolios with twenty-three points is given in Exhibit 2 for the Brazilian IBOVESPA stock index.

Finally, by construction all portfolios composing the set of efficient portfolios satisfy the following requirement: they maximize the expected return for a given level of risk (measured using R^2) as required from efficient portfolios.

4 An Example: Efficient IBOVESPA-Enhanced Portfolios

In order to illustrate the use of the methodology presented above, consider the problem of generating the set of efficient IBOVESPA-enhanced portfolios.

An approximation of twenty-three points for the set of efficient IBOVESPA-enhanced portfolios is provided in Exhibit 2. This approximation uses the eleven assets in Exhibit 3. The ten stocks in Exhibit 3 were the ones with the largest weights in the IBOVESPA for the period of analysis. Historical data covering one year (October 31, 1996, to October 31, 1997) of daily returns are used as scenarios. For this period, the annual expected return of each of the eleven assets used is given in Exhibit 3. The optimal allocation along the set of efficient IBOVESPA-enhanced portfolios is depicted in Exhibit 4.

The IBOVESPA is heavily weighted towards state-owned Brazilian companies, such as Telebrás and Petrobrás. Foreign investors cite this as a serious deficiency of the IBOVESPA (see [3] for a comparison of several Latin American stock indexes, such as the Argentinean MERVAL, the Chilean IPSA and the Mexican IPyC). For example, ordinary and preferred stocks of Telebrás (Telebrás ON and Telebrás PN, respectively) represented more than fifty-five percent (55%) of the IBOVESPA during the period of analysis. If ordinary and preferred stocks of Petrobrás (Petrobrás ON and Petrobrás PN, respectively) are added to those of Telebrás, these four stocks together represented more than seventy percent (70%) of the index during the period of analysis. This explains the high concentration depicted in Exhibit 4 on these four stocks for the largest values of R^2. Let us remember that portfolios with R^2 close to one are expected to be closely linked to the IBOVESPA.

Exhibit 3 shows that Petrobrás ON is the stock with the largest expected return for the period of analysis, followed by Petrobrás PN and Telebrás ON, in this order. This explains why the allocations on the set of efficient IBOVESPA-enhanced portfolios are concentrated on these three stocks for the smallest values of R^2, as depicted in Exhibit 4. Let us remember that portfolios with a small R^2 are those that seek to maximize the expect return of portfolio for the scenarios considered. These portfolios are not expected to be closely linked to the IBOVESPA.

For those points on the set of efficient IBOVESPA-enhanced portfolios between the extreme cases mentioned in the two paragraphs above (i.e., points with R^2 close to 0.5), it is true that:

1) The allocation concentrates on Petrobrás ON, Petrobrás PN and Telebrás ON, leaving outside all other stocks and the cash-equivalent instrument.

2) As the R^2 decreases, Telebrás PN is substituted on the set of efficient IBOVESPA-enhanced portfolios by Telebrás ON, which presents a larger expected return than Telebrás PN (see Exhibit 3), with a correlation close to one (0.95) with Telebrás PN. The same can be said about Petrobrás PN and Petrobrás ON. This illustrates that as the R^2 decreases, the model substitutes those stocks with the largest weights in the IBOVESPA by stocks with higher expected returns, but still with a high correlation with the index.

3) The allocation in cash-equivalent instruments disappears quickly as one moves from right to left on the set of efficient IBOVESPA-enhanced portfolios. This can be easily explained when analyzing Exhibit 3: the expected returns of Brazilian stocks were much higher than those of cash-equivalent instruments for the period of analysis.

As a final numerical illustration, Exhibit 5 depicts two sets of efficient IBOVESPA-enhanced portfolios: one was obtained using the ten stocks in Exhibit 3 (plus a cash-equivalent instrument), while the other was obtained using only five stocks (Telebrás PN, Petrobrás PN, Telesp PN, Eletrobrás PNB and Vale PN, plus a cash-equivalent instrument). As expected, the set of IBOVESPA-enhanced portfolios obtained using ten stocks "dominates" the set obtained using only five stocks. For example, for any fixed level of market risk (measured by the R^2), it is possible to obtain a portfolio with higher expected return when using ten stocks than when using only five stocks.

5 Conclusions

The basic principles of enhanced indexing fund management were presented in this work. A methodology to generate the set of efficient portfolios was proposed. Important operational details of the methodology for practitioners were addressed. Two of these operational details were related to market risk measurement, and the mathematical formulation of the optimization problems that need to be solved to obtain the set of efficient portfolios. The practical use of the methodology was illustrated in the case of the Brazilian IBOVESPA stock index.

References

[1] Black, F., and R.Litterman. "Asset Allocation: Combining Investor Views with Market Equilibrium." Journal of Fixed Income, 1, 7-18, 1991.

[2] Duarte, Jr., A.M. "A Comparative Study of Downside Risk and Volatility in Asset Allocation". Investigación Operativa, 4, 213-228, 1994.

[3] Duarte, Jr., A.M. "Indexing Stock Portfolios in Brazil: Tracking the IBOVESPA and the FGV100." Emerging Markets Quarterly, 1, 20-26, 1997.

[4] Elton, E.J., and M.J.Gruber. Modern Portfolio Theory and Investment Analysis. Wiley, New York, 1991.

[5] Fabozzi, F.J. (editor) Portfolio and Investment Management. Probus Publishing Company, Chicago, 1989.

[6] Fabozzi, F.J., and H.G.Fong. Advanced Fixed Income Portfolio Management: The State of the Art. Probus Publishing Company, Chicago, 1994.

[7] Fishburn, P. "Mean-Risk Analysis with Risk Associated with Below-Target Returns." American Economic Review, 67, 116-126, 1977.

[8] Koskosidis, Y.K., and A.M.Duarte, Jr. "A Scenario-Based Approach to Active Asset Allocation." Journal of Portfolio Management, 23, 74-85, 1997.

[9] Markowitz, H.M. Portfolio Selection: Efficient Diversification of Investments. Wiley: New York, 1959.

[10] Markowitz, H.M., Todd, P., Xu, G., and Yamane, Y. "Fast Computation of Mean-Variance Efficient Sets Using Historical Covariances." Journal of Financial Engineering, 1, 2, 117-132, 1992.

[11] Markowitz, H.M., Todd, P., Xu, G., and Yamane, Y. "Computation of Mean-Semivariance Efficient Sets by the Critical Line Algorithm." Annals of Operations Research, 45, 307-317, 1993.

[12] Pogue, G.A. "En Extension of the Markowitz Portfolio Selection Model to Include Variable Transaction Costs, Short Sales, Leverage Policies and Taxes." Journal of Finance, 24, 1005-1027, 1970.

[13] Rao, C.R. Linear Statistical Inference and Its Applications. Wiley: New York, 1973.

[14] Scheffé, H. The Analysis of Variance. Wiley: New York, 1959.

Exhibit 1. Set of Efficient Portfolios

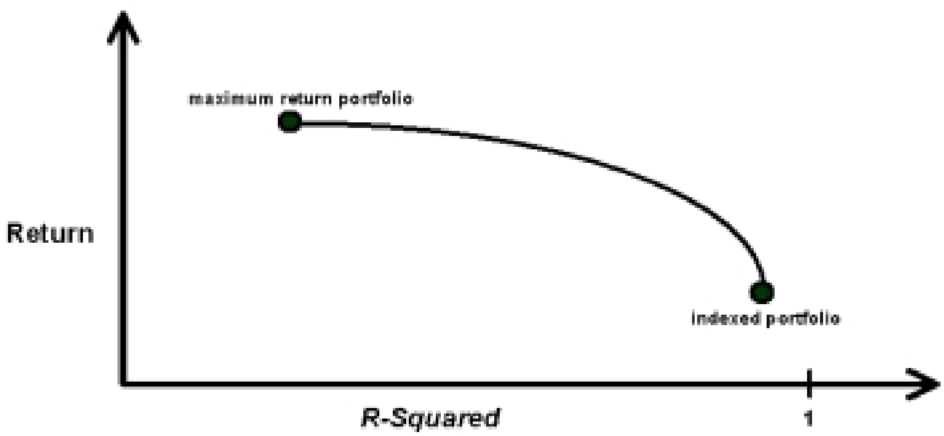

Exhibit 2. Set of Efficient IBOVESPA-Enhanced Portfolios Using Ten Stocks

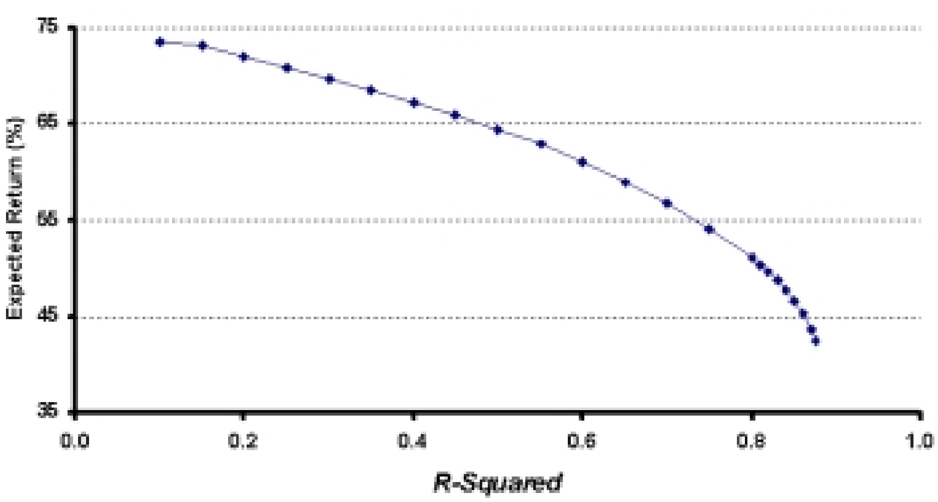

Exhibit 3. Expect Returns of Eleven Brazilian Assets

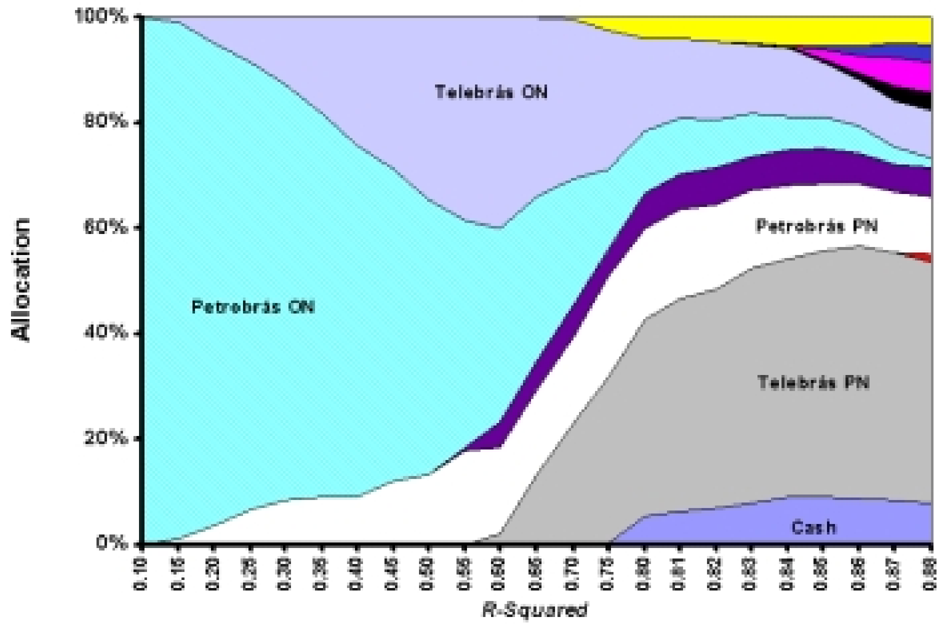

Exhibit 4. Allocation Along the Set of Efficient IBOVESPA-Enhanced Portfolios

Asset	Asset Class	Annual Expected Return
CDI-CETIP	cash equivalent	21.92%
Telebrás PN	stock	46.27%
Eletrobrás ON	stock	10.17%
Petrobrás PN	stock	60.34%
Telesp PN	stock	44.26%
Petrobrás ON	stock	73.28%
Telebrás ON	stock	52.03%
Bradesco PN	stock	18.82%
Eletrobrás PNB	stock	10.60%
Vale PN	stock	14.25%
Cemig PN	stock	48.41%

Exhibit 5. Sets of Efficient IBOVESPA-Enhanced Portfolios Using Five and Ten Stocks