# Application of a Linear Programming-based Economic Index to the Analysis of the Evolution of the Costs of Foods 

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#### Abstract

This paper develops a linear programming model able to yield optimal food shopping lists to match the nutritional needs and consumption habits of adult workers. The results obtained are used to introduce and evaluate an index with which it was possible to analyze the evolution of the food costs in the Municipality of Rio de Janeiro, from January 1996 to March 1997, and compare them to the results calculated by DIEESE.


Keywords: Linear Programming, Cost Indices.

## 1 Introduction

The aim of this paper is to introduce an index which we shall denominate Índice de Custo dos Alimentos do Município do Rio de Janeiro (Food Cost Index for the Municipality of Rio de Janeiro) or ICA-RJ that is based on a linear programming model suggested by Lins $[9,10]$ and to compare it to the one evaluated by DIEESE, namely, the Índice Estatistico de Preços (Statistical Price Index).

DIEESE, an acronym which for Departamento Intersindical de Estatística e Estudos SócioEconômicos (a Labor Union department), has established a methodology of the evolution of the costs of a given essential food shopping list [2]. This survey is carried out in fourteen Brazilian state capitals and monitors on a monthly basis the evolution of the costs of a determined food shopping list consisting of thirteen food entries which was written down by Governmental Ordinance \# 399 which regulated the minimum wage in Brazil.

In Brazil there is an economic index, which is also based on a linear programming model. This index is denominated Índice Nacional do SINAPI (or the short SINAPI index) developed by Lins, put to trial by the Banco Nacional de Habitação - BNH in 1969 and since then successfully applied. Today the survey concerning this index is carried out in all states of the Brazil [1].

In section 2 we describe the statistical price index methodology employed by DIEESE and its historical importance; in section 3 we show the Food Cost Index, its formulation, the optimization model and we present the results obtained; in section 4 we compare the results obtained by the methodologies cited above; in section 5 we underline the development possibilities of our methodology; in section 6 we present the conclusion and complements; and in section 7 we suggest other applications of our optimization model.

## 2 Statistical Price index Methodology Employed by DIEESE

DIEESE has established the methodology for the evolution of a given list of essential foods which is denominated Minimum Essential Ration [2]. The survey on the Minimum Essential Ration carried out in fourteen Brazilian state capitals monitors monthly the evolution of the prices concerning a fixed list of essential foods which consists of thirteen items as well as the monthly expenditure that a worker would have in order to buy those essential foods. Another information given by this survey is the number of necessary working hours such that an individual who earns the minimum wage may afford the purchasing of those foods. The necessary minimum wage, which is also monthly evaluated, is calculated with respect to the monthly expenditure on foods and this information is collected when surveying the Minimum Essential Ration.

The methodology employed by DIEESE for the survey on the Minimum Essential Ration was established with respect to the Governmental Ordinance \# 399 that regulates the minimum wage in Brazil. On April 30, 1938, Federal Law \# 185 of January 14, 1936, was given detailed regulations on its application by the Governmental Ordinance \# 399. That ordinance established that the minimum wage is the payment due to an adult worker, without gender bias, per ordinary working day, capable of the satisfying in a given period time and region of the country the usual needs of food, housing, clothing, health and transport. By census studying taken in each Brazilian city and through data concerning wages
obtained from the commerce and industry of several regions, the Minimum Wage Commissions, which were created before the institutionalization of Ordinance \# 399, established the minimum regional values to be paid to blue-collar workers. They have also presented a list of foods together with their respective minimum quantities. That list of foods was christened the Minimum Essential Ration and it should be sufficient for the keeping and well-being of an adult worker and should also contain stipulated quantities are differentiated by regions, as the following Table 1 shows.

The Procedure that was employed in each state capital where those surveys were carried out was described by Lucas [11].

### 2.1 Calculation of the Monthly Cost of the Minimum Essential Ration

Each month, after the price survey, the average prices of foods are calculated. These averages are taken according to the kind of store (supermarket, general store, butcher's and baker's, etc.) where the essential foods are purchased by using the following procedure for each one of the surveyed products:
(a) An arithmetic mean for all prices surveyed at the purchasing spots is calculated so as to obtain an average related to the type of store where foods were purchased (for each food).
(b) The same procedure is employed with respect to food purchased in other stores.
(c) The various results are added and therefore we obtain the weighted average price of a particular item.

The average price of each food multiplied by the quantities defined by Governmental Ordinance \# 399 indicates the monthly expenditure of a worker with the purchasing of the Minimum Essential Ration.

### 2.2 Evaluation of Monthly Variation of the Minimum Essential Ration

Actually, the monthly variation in the Minimum Essential Ration publish by DIEESE consists of a variation which is relative to the preceding month. Its evaluation is performed according to the following formula:
$\mathrm{V} r_{t-1, t}=\frac{R_{t}-R_{t-1}}{R_{t-1}}$
where $\operatorname{Vr}{ }_{t-1, t}$ is the Minimum Essential Ration relative cost variation between month $t$ and the preceding month $t-1$, $t$ is the reference month for this survey, $R_{t}$ is the Minimum Essential Ration cost in month $t$ and $R_{t-1}$ is the Minimum Essential

Ration cost in month t-1.

| FOODS | REGION I | REGION 2 | REGION 3 | NATIONAL |
| :--- | :--- | :--- | :--- | :--- |
| Beef | 6.0 kg | 4.5 kg | 6.6 kg | 6.0 kg |
| Milk | 7.51 | 6.01 | 7.51 | 15.01 |
| Beans | 4.5 kg | 4.5 kg | 4.5 kg | 4.5 kg |
| Rice | 3.0 kg | 3.6 kg | 3.0 kg | 3.0 kg |
| Manioc Flour | 1.5 kg | 3.0 kg | 1.5 kg | 1.5 kg |
| Potato | 6.0 kg | I. | 6.0 kg | 6.0 kg |
| Tomato | 9.0 kg | 12.0 kg | 9.0 kg | 9.0 kg |
| Bread | 6.0 kg | 6.0 kg | 6.0 kg | 6.0 kg |
| Coffe | 0.6 kg | 300 g | 0.6 kg | 0.6 kg |
| Banana | 7.5 dz | 90 u | 90 u | 90 u |
| Sugar | 3.0 kg | 3.0 kg | 3.0 kg | 3.0 kg |
| Soy Oil | 900 ml | 750 ml | 900 ml | 1.51 |
| Butter | 0.75 kg | 0.75 kg | 0.75 kg | 0.90 kg |

Table 1: Minimum Essential Ration for each region

In the above table the regions are defined as follows:
Region 1- Comprises the states of São Paulo, Minas Gerais, Espírito Santo and Rio de Janeiro

Region 2 - Comprises the states of Pernambuco, Bahia, Ceará, Rio Grande do Norte, Alagoas, Sergipe, Amazonas, Pará, Piaui, Goiás, Acre, Paraiba, Rondônia, Amapá and Roraima.

Region 3 - Comprises the states of Paraná, Santa Catarina, Rio Grande do Sul, Mato Grosso e Mato Grosso do Sul.

National - Defined by the Average Standard Ration for the working population in diverse employment in the whole of Brazil.

### 2.3 Study of Variation of Minimum Essential Ration in the Municipality of Rio de Janeiro from January 1996 to March 1997

The DIEESE office in Rio de Janeiro carries out a monthly survey concerning the prices of basic food in the Municipality of Rio de Janeiro and sends the results to the São Paulo office where national information are centralized and processed. The Table 2 shows the Minimum Essential Ration and their prices for the Municipality of Rio de Janeiro.

| F00105 | QUANTITIES | Price Evolution of the DIEFSEIs cakubation Janearyivo to March/y? Beferenes.Manth - Minimum Esacctial Bution |  |  |
| :---: | :---: | :---: | :---: | :---: |
| BEEF | 6.018 |  |  |  |
| MILK | 751 | Fablas | A\$ | हく2, 24 |
| BEANS | 4.5 kg | Man'95 | R5 | B1,11 |
| RKCE | 3.0kE | Apri96 | R3 | 80, 80 |
| MANTOC FLOUR | 1.5 kg | May996 | R5 | 87.24 |
| PCTATO | 6.0 kg | -un96 | R3 | 68,82 85.69 |
| TOMATO | 9.01 kg | Aug's | As | 68, 47 |
| BरEAD | 6.0 kz | Sap/26 | R3 | 84,21 |
| COFFE | 0.6 kg | Dcras | H5 | 18, 03 |
| BANANA | 75 dc | Now36 | R\$ | 83,92 |
| SLCAR | 3.0 kz | Decog dan 27 | RS | 84.13 85.28 |
| SOY OIL | 900 nl | Febrer | AS | 80.78 |
| BUITER | 0.75 kg | Maraz | RS | 92.88 |

Table 2: The Minimum Essential Ration and their Price Evolution (Rio de Janeiro-Jan/96 to Mar/97)

The price of the Minimum Essential Ration has evolved with marked oscillations as shows in the Figure 1.


Figure 1: Price Evolution of the Minimum Essential Ration

## 3 The Food Cost Index for the Municipality of Rio de Janeiro

The basic idea behind the Food Cost Index is to follow up the cost evolution of a set of Foods that can change in time [9,10]. The set of food items considered with respect to each reference month will be the optimal solution of the linear programming model built specifically for this case. The optimization model considers the nutritional
needs of adult workers and their consumption habits. The geographical span of the index is limited to the Municipality of Rio de Janeiro, since Brazil's continental dimensions allow for consumption habits which are as diverse as the number of different regions in this country. The obtained results relate to the time period that starts in January 1996 and ends in March 1997.

### 3.1 Mathematical Formulation of the Food Cost Index

We define the Food Cost Index by the relation:
$\mathrm{ICA}_{t-1, t}=\frac{q *^{t} \cdot p^{t}}{q *^{t-1} \cdot p^{t-1}}$
where $\mathrm{ICA}_{t-1, t}$ is the measure of the monthly variation of the cost of foods between the reference month $t$ and the previous month $t-1$. The vectors $\mathrm{q}^{* t}$ and $\mathrm{q}^{* t-1}$ are vectors which represent food items allowances for the months $t$ and $t-1$, respectively, and are the optimal solution of the optimization model. The $\mathrm{p}^{t}$ and $\mathrm{p}^{t-1}$ are the foods price vectors for months above.

### 3.2 The Optimization Model

The rationale behind the building of an optimization model in order to choose a basic set of foods for each reference month is to guarantee with certainty that set of foods and their respective amounts, is capable of satisfying a worker's nutritional needs and at the same time waiting on theirs feeding habits, with the least cost. The quantity vector

$$
\mathrm{q}^{t}=\left[q_{1^{t}}, q_{2^{t}}, \ldots, q_{n^{t}}\right]
$$

where $\mathrm{i}=1,2, \ldots, 70,71$, is such that it minimizes the global cost:

$$
\begin{equation*}
\mathrm{C}^{t}=\left(\mathrm{q}^{t} \cdot \mathrm{p}^{t}\right) \tag{3}
\end{equation*}
$$

subject to the following constraints:
$\left[\mathrm{A}^{t}\right] \times\left[\mathrm{q}^{t}\right] \geq[\mathrm{N}]$, (nutritional needs)
$\left[\mathrm{S}^{t}\right] \times\left[\mathrm{q}^{t}\right] \geq[\mathrm{K}]$, (consumption habits)
$\left[\mathrm{U}^{t}\right] \times\left[\mathrm{q}^{t}\right] \leq[\mathrm{L}]$ (consumption habits)
and $\mathrm{q}_{i^{t}} \geq 0$

Here $\left[\mathrm{A}^{t}\right]$ is the matrix of the nutritional needs; $[\mathrm{N}]$ is the vector of the components of which inform us the lower bounds of a worker's nutritional needs during 30 days: $\left[\mathrm{S}^{t}\right]$ and $\left[\mathrm{U}^{t}\right]$ are matrices the entries of which reflect the consumption habits in the geographical region in question- the vectors [K] and [L] give information concerning a worker's feeding tastes. In this work we have built this model, monthly changed the information concerning the food costs and obtained the optimal food shopping lists.

### 3.3 Evaluation of Production Costs of Foods in the Model

After a considerable amount of surveying, we chose DESIP, an IBGE department that concerns itself with price indexes. IBGE stands for Instituto Brasileiro de Geografia e Estatística, a traditional Brazilian institution that is devoted to geographical and social statistical questions. IBGE fixes on a monthly basis the INPC - Indice Nacional de Preços para o Consumidor (Consumer's National Price Index) which has a mathematical weight structure built in with differentiated aggregation levels [5].

The structures are built according to a label classification and form groups which are established in a logical way so that consumption categories of the same nature go together. This results in the following levels of aggregation:

## Group <br> Subgroup

Item

## Sub-item

As an example, the item Orange is a sub-item of the item Fruits which, along with other items, belongs to the subgroup Domestic Nutrition which together with subgroup Non-domestic Nutrition, belong to the group Food and Beverages. Here we have utilized information available at the subgroup Food level concerning the Municipality of Rio de Janeiro. Initially we obtained from DESIP/IBGE a general list of prices referring to September 95 [11] and in order to recover the prices of each item in the necessary time period we have utilized information on the monthly variation indexes of each item ( $\mathrm{V}_{t, t-1}$ ) in the Municipality of Rio de Janeiro available at SIDRA data bank of IBGE in html format $[4,5]$. The mathematical formula employed by DESIP in the evaluation of the relative monthly variation is the following[5]:
$\mathrm{V}_{t, t-1}=\left(\mathrm{R}_{t-1, t}^{j}\right) \times 100$,
where $R t, t-1$ is the measure of the price variation on the food $j$ between months $t-1$, the preceding month, and t the reference month and is defined by:

$$
\begin{equation*}
\mathrm{R}_{t-1, t}^{j}=\frac{p_{t}^{j}}{p_{t-1}^{j}} \tag{9}
\end{equation*}
$$

where $\mathrm{p}_{t}^{j}$ is the average price of food $j$ in the month t and $\mathrm{p}_{t-1}^{j}$ means the same but for the preceding month. Since we had a list of basic prices and had at
our disposal monthly variation of each sub-item, it was possible to recompose the monthly price of each food. The formula that we employed in this re-composition was:
$\mathrm{p}_{t}^{j}=\mathrm{p}_{t-1}^{j}\left(\frac{V_{t, t-1}}{100}+1\right)$
In this way it was possible to obtain average prices of all foods in the period from September 95 to May 97 [11].

### 3.4 Modeling of the Nutritional Information

We have determined the nutrients to be considered in the model following the guidance of team of nutritionist from the Instituto de Nutrição da UFRJ who are specialists in natural nutrition. The nutrients selected are: Calorie, Protein, Calcium, Phosphorus, Iron and the Vitamins A, Bl, B2, PP, C.

The first constraint set, see equation (4), assures that the optimal food shopping list chosen by the model satisfies the nutritional needs of an adult worker, man or woman, and in order to accomplish this it was also necessary to obtain the nutritional composition formula of each food in the model. At this stage we have consulted the table of nutrient composition published by IBGE [6], since this source lists the best available bibliographical references. We based the model on 71 foods where both the price and the nutritional value were known (Table 3).

Through the Food and Nutrition Board [12] we have obtained information on the minimum nutrient quantities which a workers needs in order to subsist for a period of 30 days. In order to define the nutritional parameters that are used to establish the lower bounds for the nutrients, we adopted a rule according to which we choose for each nutrient the greatest requested amount for ages above 15 years old to guarantee that the optimal food shopping lists yielded by the model satisfies the needs of any worker, man or woman, above this age. A human being has varied nutritional needs during his or her lifetime. For example, consider the nutrient Iron. Among ages above 15 years old, women between 25 and 50 years old are the ones that are in most need of this nutrient. Those women daily need 15 grams , therefore we have chosen this figure as the lower bound of Iron consumption as it satisfies also men and women that are older than 15 years old.

### 3.5 Modeling of Data Concerning the Consumer's Habits

In order to define the parameters of the model which assure the keeping of consumer's habits we have drawn on information concerning the Minimum Essential Ration produced by DIEESE [2].

The DIEESE Minimum Essential Ration is constituted by 13 items and was defined by an analysis of a census taken some time ago [2]. Hence we endeavored

| CODE | FOOD | CODE | FOOD | CODE | FOOD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A 1 | Rice | A 25 | Orange Type 1 | A 49 | Eggs |
| A2 | Black Beans | A 26 | Orange Type2 | A 50 | Mik |
| A3 | Pasta | A 27 | Apple | A 51 | Dehydr. Mik |
| A 4 | Corn Flour | A 28 | Papaya | A 52 | Yougurt |
| A5 | Wheat Flour | A 29 | Pear | A 53 | CheeseType1 |
| A 6 | Manioc Flour | A 30 | Grape | A 54 | CheeseType? |
| A 7 | Potato | A 31 | Orange Type3 | A 55 | Cabbage |
| $A B$ | Pumpkin | A 32 | Liver | A 56 | Butter |
| A9 | Choyote | A 33 | Pork Meat | A 57 | Bread Type1 |
| A 10 | Green pepper | A 34 | Meat Type 1 | A 58 | Bread Type2 |
| A 11 | Okra | A 35 | Meat Type 2 | A 59 | Cake |
| A 12 | Tomato | A 36 | Meat Type 3 | A 60 | Soy Oil |
| A 13 | Onion | A 37 | Meat Type 4 | A61 | Margerine |
| A 14 | Carrot | A 38 | Meat Type 5 | A 62 | Coftee |
| A 15 | Sugar | A 39 | Meat Type 6 | A 63 | Soft Drinks |
| A 16 | Letuce | A 40 | Meat Type 7 | A 64 | Beer |
| A 17 | Kale | A 41 | Fish 1 | A 65 | Canned Pea |
| A 18 | Cauliflower | A 42 | Fish 2 | A 66 | Canned Sardine |
| A 19 | Cabbage | A 43 | Fish 3 | A 67 | Canned Sausage |
| A 20 | Season Herb | A 44 | Sardine | A 68 | Tomato Sause |
| A 21 | Watercress | A 45 | Fish 4 | A 69 | All |
| A 22 | Ananas | A 46 | Pork Sausage | A 70 | Salt |
| A 23 | BananaType1 | A 47 | Cod | A 71 | Maionnese |
| A 24 | BananaType2 | A 48 | Chicken |  |  |

Table 3: Food List in the Model


Table 4: Lower Bounds of the Nutrients
to fit the 71 food items that we have chosen into the greater groups also employed by DIEESE and take advantage of the parameters obtained in the above-mentioned census. In those cases where this procedure could not be applied we had recourse to the POF / IBGE [7]. Here POF stands for Pesquisa de Orçamento Familiar
(domestic budget survey). While endeavoring to achieve the greatest possible level of coherence, we should stress that it is based on an assumption of consumer's habits since there was never a specific field survey concerning a project of this kind. The Table 5 describes the methodology. Notice that qi is the amount of food item i in the measure units employed.


Table 5: An example from the Tubers, Roots and Vegetables group.

As one can notice, we have utilized a $\pm 10 \%$ interval for the value of the DIEESE food group for the lower and upper bounds of the equivalent food group, except for the group Canned Food and Beverages for which we fixed zero as a lower bound, so as not to maintain those items in the basic lists.

In this paper the Tubers, Roots and Vegetables group is formed by 8 foods (potato, pumpkin, choyote, green pepper, okra, tomato, onion and carrot) in which the sum of all quantities chosen by the model are within the interval described above. The same procedure was employed for the rest of the groups [11].

### 3.6 Complementary Constraints

When we implemented the model with the nutritional constraints and the constraints described in the last table, we observed interesting facts that led us to intervene in the way described below.

In the Cereals, Leguminous and Oleaginous Group the model has chosen all the available quantity for Bean only. We have opted to add constraints in order to guarantee the presence of the item Rice in the list in amounts $\pm 10 \%$ of the available amount present in Minimum Essential Ration and this produced the lower and the upper bound 2.7 kg and 3.3 kg respectively. (This food has a large popular preference in its group).

There was a total concentration of the Tubers, Roots and Vegetables group on a single food item. We have allowed for a maximum of 3.0 kg for each food item (since this represented $50 \%$ of the amount allocated to the item potato in Minimum Essential Ration, so that some variation would be presented in optimal food shopping list (at least variation the same as Minimum Essential Ration), with the sole exception of the item onion, amounts of which were limited to 0.50 kg ,
according to the POF survey [7].
There was a total concentration of Meat, Viscera, Fish, Industrializes Meat and Fish, Fowl and Eggs group on the item Bovine Liver as in the developed by Stigler [13]. To obtain a better fit between the model and the consumption habits we have imposed the additional condition that the sum of the food amounts in the group without viscera had to be equal to the lower bound for the group as a whole, which is 5.4 kg .

We observed that the food item Salt was always present with its upper bound. Therefore we had recourse to POF survey [7] to establish a upper bound of 0.36 kg . (Because this food has calcium and is inexpensive).

### 3.7 Optimal Food Shopping Lists

In order to obtain optimal food shopping lists for each reference month together with their respective costs we have implemented a model for each reference month on which constraints were imposed maintained and the values of the objective function were updated. The optimal solution for each reference month solutions were found in an average time of 0.06 sec . by CPLEX Linear Optimization 3. 0 software. Part of the software report were selected and can be found in Lucas [11].

## Work Station Description

Host: pol.pep.ufrj.br

Model: Digital - DEC 3000

Processor: alpha 125

System: OSF/I 3.0
RAM Memory: 32 Mb
Swap: 324 Mb
Disk: 2.5 Gb
Notice that the optimal food lists were modified in the period studied reflecting seasonal effect changes and that some food groups such as Fruit group concentrate its needs on a single food entry (see Table 6 and Table 7). In the group Cereals and Vegetables this did not happen due to the fact that we have included constraints that guaranteed variation within the group. It is worthwhile to point out that in June 1996 (Table 6) the model substituted totally the item French Bread for Cream Cracker Biscuits and this is due to a $4.6 \%$ increase in the price of the item French Bread which was followed by an increase of the item Cream Crackers Biscuits in the
months of July 1996 and August. A substitution this huge was not forecasted since the entry French Bread did not exhibit a lower bound.

| Cove | Foods | Jan96 | FobV96 | Marte | Apm93 | May98 | Jum9 | Jumb |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A1 | RICE | 2.0 | 2,70 | 2,70 | 2.0 | 270 | 2,70 | 2.70 |
| N | Beans | 4.05 | 4,85 | 4.98 | 4.05 | 4.05 | 4.05 | 4.05 |
| A ${ }^{3}$ | MAN. FLOUR | 1,35 | 1,35 | 1,35 | 1,35 | 1.35 | 1,35 | 1,35 |
| AT | POTATO | 300 | 3.00 | 3.00 | 309 | 300 | 3.00 | 209 |
| 48 | PUMPKIN | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 20 | CHAYOTE | 3.00 | 3,00 | 3.00 | 3.00 | 3.00 | 3,00 | 200 |
| A18 | TOMATO |  | - | - | 1,00 | 1.00 | - | 1.09 |
| A11 | OKRA | 3,00 | 3,00 | 3.00 | . | . | 1,00 | , |
| A13 | ONIOM | 0,50 | 0,50 | 0.50 | 0,50 | 0.50 | 0,50 | 0.50 |
| A14 | CARROT | 1.00 | 1,90 | 1.00 | 300 | 3.09 | 3.00 | 3.00 |
| A15 | SUGAR | 2,70 | 2,70 | 2,70 | 2,70 | 270 | 2,70 | 2.70 |
| A18 | Cabbace | 0.43 | 0,43 | 0.43 | 0.4. | 0.43 | 0.43 | 0.43 |
| A31 | ORANGE I | 6.75 | 6.75 | 6.75 | 675 | 6.75 | 675 | . |
| A25 | ORANGE II | * | , | * | , | * | . | 6.75 |
| 235 | ORANQE II | - | * | * | - | * | - | - |
| A25 | PAPAYA | - | ${ }^{-}$ | - | - | - | - | - |
| $\mathrm{AsF}^{2}$ | UVER | 0,16 | 0,16 | 0.19 | 0,22 | 0.22 | 0,00 | 0,17 |
| M4 4 | SARDINE | - | - | 0.41 | a.30 | 0.3s | 1,77 | 1.63 |
| A45 | CHICKEN | 4,80 | 4,90 | 4.80 | 4,80 | 4.80 | 4,80 | 4,80 |
| AM | Ega | 0.44 | 0,44 | * | . | . | . | . |
| ${ }^{4} 59$ | MLx | 6.17 | 6.17 | 6.27 | 5,86 | 5.85 | 6.18 | 6.38 |
| AS3 | u. CHEESE | 2,66 | 2,08 | 1,99 | 2,30 | 2.30 | 1,43 | 1,87 |
| Ass | BUTTER | 0,68 | 0,88 | 0.64 | 9,6a | 0.64 | 0,68 | 0,68 |
| A57 | BREAD | 5.40 | 5,40 | 5.69 | 5,40 | 5.49 | - | 5.50 |
| ASS | CEEASA | - | - | * | * | * | 5,40 | - |
|  | CRAKER mis |  |  |  |  |  |  |  |
| ABS | soy Oll. | 0,81 | 0.91 | 0.81 | 0.81 | 0.81 | 0.81 | 0.81 |
| AE2 | corree | 0.54 | 0.54 | 0.54 | 0,54 | 0.54 | 0,54 | 0.54 |
| A63 | TOMATO SAUCE | 0.22 | 0.22 | a.22 | 0.28 | 0.22 | 0.28 | 0.22 |
| aro | SALT | 0,20 | 0.20 | 0.20 | 0,20 | 0,20 | 0.20 | 0,20 |
|  | $\operatorname{cost}$ (R9) | 67.00 | 66.45 | 60.16 | 67.00 | 68.30 | 69.22 | 70,30 |

Table 6

## 4 Comparison between Results Obtained with Both Methodologies

### 4.1 Cost Evolution Analysis

The ICA-RJ Optimal Food Shopping List was found to be at a lower level when compared to the DIEESE Methodology (see Figure 2). This difference has occurred mainly because our model chose the item Chicken to form the Optimal Food

| Code | Foods | Ang 76 | Sepree | Octre | Nowas | Dece9s | Jemay | Fubery | Nav97 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 41 | nice | 2,70 | 2,70 | 2.70 | 2.0 | 2.70 | 270 | 2.70 | 2,70 |
| 12 | BEANS | 4,05 | 4,05 | 4.05 | 4.05 | 4.95 | 4,06 | 4.05 | 4,05 |
| A 18 | ULew. FLOUP | 1,35 | 1,35 | 1.35 | 1.35 | 1,32 | 1,35 | 1.35 | 1,35 |
| AT | POTATO | 3.09 | 3.09 | 100 | 3.00 | 3,00 | 2,60 | 3.00 | 3.00 |
| 48 | Pumpion | 3,00 | 3,00 | 1,00 | 3,00 | 3.00 | 360 | 3.99 | 8 Mo |
| $N$ | chayote | 3,00 | 3.00 | 3.00 | 3.00 | 3.90 | 300 | 1.09 | 3.00 |
| 12 | томато | 1.05 | 1,00 | 3.07 | 1.90 | 1,90 | 100 | 1,00 | - |
| A11 | CNCRA | - | - | . | . | . | 1.00 | 3.00 | 1,00 |
| A13 | conom | 0,00 | 0,50 | 0,50 | 0.50 | 0.50 | 0.50 | 0.89 | 0.50 |
| A14 | cannot | 3.00 | 8.00 | 3.08 | 3.09 | 3.00 | . | - | 3,00 |
| A15 | 8ucas | 2.0 | 2,70 | 2,70 | 2,70 | 2,70 | 2.00 | 2,\% | 2,70 |
| 48 | canowat | 0.43 | 0,43 | 0,43 | 0,43 | 0.43 | 0.49 | 6,43 | 0.43 |
| $\times 1$ | omancei | . | , | . | . | . | . | 6.75 | 5.75 |
| 228 | crameeil | 6.75 | - | 6.75 | - | 5,75 | 4,75 | - | - |
| 125 | CRange ili | - | 5.75 | - | - | - |  | * | * |
| 228 | PAPAYA | - | - | . | 6.75 | - | - | - | - |
| 1.12 | UVER | 0,17 | 0.24 | 021 | 0.45 | 0.17 | 0.21 | 0,18 | 2.21 |
| 2.4 | SARCNE | 1,43 | 0.36 | 0.39 | 1,48 | 1,83 | 0,33 | 0.41 | 0.80 |
| 448 | chacken | 4,90 | 4,80 | 4,30 | 4,30 | 4,80 | 4,95 | 4.80 | 4.80 |
| M ${ }^{8}$ | eng | - | . | - | - | - | - | - | - |
| ASD | MEK | 6.38 | 5, 22 | 5.93 | 6,22 | 6.30 | 5,93 | 8,21 | 6.00 |
| 458 | m. Cherse | 1, 17 | 2.43 | 2,32 | 2,03 | 1.87 | 2,32 | 2.06 | 2.16 |
| A55 | nutter | 0,68 | 0.68 | 0.68 | 0,68 | 0.68 | 6.98 | 0.68 | 9.98 |
| MST | becas | 5.40 | 5.40 | 5.40 | 5.40 | 5.40 | 5,49 | 5,40 | 8, 20 |
| Ass | CREAM CRACKER BIS | - | - | - | - | - | - | . | . |
| A50 | SOY CIL | 0.81 | 0.81 | 0.81 | 0,81 | 0.81 | 0.91 | 0.81 | 0.81 |
| A12 | corree | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 |
| M50 | TOLATO Seute | 0,22 | 0.22 | 0.28 | 0.28 | 0.22 | 0,22 | 0.22 | 0.22 |
| A 70 | 8ALT | 0.80 | 0.80 | 0.20 | 2,20 | 0.20 | 0.20 | 0.20 | 0.20 |
| Cont (\%) |  | 75.80 | 66,98 | 69.55 | 45,24 | e2, 82 | 70,73 | 71,13 | 73,51 |

Table 7

Shopping Lists during all the time period under study.

On the other hand DIEESE fixes the item Beef in its Minimum Essential Ration.

### 4.2 Analysis of the Monthly Indices Yielded by the Two Methodologies

The Minimum Essential Ration issued by DIEESE showed bigger oscillations than the one issued by ICA-RJ (see Figure 3 and 4) reaching the level of $6.80 \%$ in March 1997 due mainly to a $58.72 \%$ increase in the price of the entry tomato [11], since there is no information on seasonal variations in the method employed by DIEESE.


Figure 2: Comparison of the Prices


Figure 3: Comparison between the Monthly Indices

### 4.3 Comparison between the Two Accumulative Indices (12 Months)

The accumulated indices of the Minimum Essential Ration issued by DIEESE are seen to form a sequence with greater monthly increases than the accumulated indices given by the method employed by the ICA-RJ (see Figure 5).

Annual Cumulative Indices

## 5 Development Possibilities

The model developed in this paper satisfies, from the theoretical point of view, the consumption habits of the workers in the Municipality of Rio de Janeiro, but as we


Figure 4: Comparison between the monthly variations related to the two methodologies


Figure 5: Comparison between the Accumulative Indices
have mentioned before, we have not taken advantage of a specific survey for a model of this kind. Below we present some important considerations:

We suggest a survey in which the amounts of foods are defined by modeling of preference representative of the real basic food list. For example, in the case of a
dietary reduction in the amounts of rice item what would be its substitute? Pasta? Manioc Flour? Or some other foods? And what about the reduction levels?

We propose that the basic-food stocks which the government regulates, are included in the model in order to reflect inflation in the period studied. And in the event of an increase in costs this should be ascribed to the proper index, for example: rice, bean, coffee, sugar, bread among others.

We indicate the inclusion of upper bounds for all foods to prevent any distortion as happened in June 96 when the model totally substituted bread with cream cracker biscuits and this food would not be enough to meet a demand of this importance.

## 6 Conclusion

The methodology employed by DIEESE in the evolution of the price of the Minimum Essential Ration considers a fixed food list, hence seasonal effects are not considered. At this point, a food cost index (ICA) is an important advantage since, besides including the seasonal effects it also allows for a modification in the basic food list without a survey. In the evaluation of IPCA index, DESIP/IBGE allows for a seasonal adjustment of the foods traditionally affected [8]. In order to obtain this correction the Paasche Index [3,4] is employed which allows for amount variations with time. This certainly is an efficient technical way of solving the problem. However, unexpected outcomes due to environmental problems, for example, may not be forecasted in advance and this would result in undue inflation, since prices go up with shortage and demand (the amount prescribed by Basic Food List does not change in order to reflect this storage). But in the optimization model, any kind of change, even an unexpected one, that brings about an increase in the cost of a given food is immediately incorporated in the model, so that one or more items are changed, fact that reflects the consumer's intelligence.

Hence, we conclude that the cost index elaborated in this paper is flexible enough to pick up information from costs as well as rigid enough to achieve its aims, becoming then an efficient index to follow up the evolution of the food costs.

## 7 Other Application Suggestions

The model developed in this work could also be implemented in the following:
Corporations - In order to benefit employees with a basic foods shopping list that satisfies their nutritional needs as well as their consumption habits. Employees could be surveyed in order to achieve a complete result.

Philanthropic institutions - Would be possible to institutions that operate with a minimum amount of financial resources to help those in need with a really efficient basic foods.

Governments - In order to subsidize consumer guidance policies which have the purpose of improve the feeding habits of the population.

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