A Continuous Approximation Model for Vehicle Routing in Solid Waste Management Systems

Octavio A. Carranza Torres[1] *Francesc Robusté Antón*[2]

[1]Universidad de San André
Business Administration Department
Vito Dumas 284-1644
Victoria, Argentina
Fax:(54-1) 746-2608
ocarranz@udesa.edu.ar

[2]Universidad Politécnica de Catalunya

Abstract
The problem of designing routes for solid waste collection systems is analyzed in the "house to house" variant. Marginal costs area,-according to route distances from depot- are determined to propose optimal sizes of zones. A continuous approximation approach is used to propose simple formulas to design routes in square grid metrics with alternative senses. A methodology is proposed with partial results applied to a real case, "Students-City", whose name is maintained anonymous. The problem of inside odd nodes within oriented networks is analyzed. Guidelines for a generic classification of networks are given and a hypothetical formula proposed for any type of network.

1 Scope and Objective

Waste Management has evolved as a complex discipline in the last decades. One key subsystem of solid waste management is designing vehicle routes with all the realistic constraints (geographical, technological, labor related, etc.). The basic problem consists of collecting garbage "house by house" and, therefore "street by street"; in vehicle routing this problem is known as Chinese Postman Problem (CPP). Even though selective collection and the introduction of garbage containers are being increasingly used –a problem that requires a rather different solution-, the CPP is a general operating rule in Latin America and may be useful for a variety of instances and cities.

The objective is to obtain general and practical recommendations for designing urban garbage collection systems through a simple approach. A manual method is proposed.

This paper is divided in six more sections. Section two reviews the antecedents of the problem, the literature found, describes the particular characteristics of the problem, formally presents the case of "Students-city" and analyzes some findings of the analytic methodology that will be used. In the third section slenderness formulas are deduced for CPP and grids with regular characteristics applying continous approximations. In the fourth section a methodology for the design of operation schemes is proposed generating results for "Students-City". In section five the problem of grids with interior nodes and orientations of arcs that are different from the classical alternative ways in square grids is analyzed. The last section discusses the results of the investigation and proposes new lines of research.

2 Antecedents

2.1 Literature Review

Two optimization problems are related to the subject: Specifically, the analysis of the chinese postman problem has addressed in the past the waste collection problem. The design of routes has been studied -refering to the generation of routes between cities- by the vehicle routing problem (VRP).

The CPP has been stated as finding a Euler tour in a graph (Berge,1966). The existence of this tour depends on the degree of each of the nodes in the graph. The solution of the CPP by creating an Euler graph (converting a graph with no Euler tour into a one with an Euler tour) has received early attention. Mei-Ko (1962) proposes a rule to minimize the distance required to cover every link in a graph with 2n odd nodes: n paths must be duplicated between the nodes such that the total length of links visited more than once is minimum. Glover (1967) uses this procedure to create pseudo-edges which are selected iteratively. Murty (1967) solves a symmetric assignment problem whose cost matrix is made up of the lengths of the shortest paths between every pair of odd nodes. Lin and Zhao (1988) propose an algorithm to solve the CPP based on a linear programming formulation.

More practical approaches can be found. Stricker (1969) proposes a decomposition algorithm -a modification of Murty's- to solve the M-Chinese Postman Problem, which arises when a graph has to be partitioned in subsets because of capacity restrictions; he applies his algorithm to the city of Cambridge. Beltrami and Bodin (1974) make a review of the problems found in New York' s Environmental Protection Agency, routing problems, that authors divide in link-routing problems and node-routing problems: the CPP may be assimilated to the first type of problem and the Traveling salesman to the latter. Bodin and Kursh (1979) present an algorithm to route mechanic sweepers, there' s an additional restriction to the CPP described by Beltrami and Bodin: While in their case they had to collect garbage home by home, in this one they have to consider parking restrictions. Bodin et al (1989) propose a software package for vehicle routing in a city of 327,000 inhabitants. Alvarez Valdes et al. (1993) propose a system to design routes in a city of 60,000 inhabitants. Evans and Minieka (1992) present a survey of arc routing methods applied to the solution of problems in waste disposal; the CPP in directed , undirected, and mixed graphs various optimum solutions.

An interesting approach to the CPP for our purpose is that presented by Larson and Odoni (1980). They focus on the fact that sufficiently good (and manual) solutions to the CPP can be obtained with the help of a map. The total length of CPP is the sum of the arcs of the graph plus the length of a the minimum length matching of odd degree pairs of nodes that may exist in the graph. The analysis of the existence and location of odd nodes is essential for this purpose.

The generation of routes and the VRP have received early attention in the literature. Three well-known heuristcs are Gillet and Miller's (1971) sweep algorithm, Clarke and Wright's (1964) savings algorithm and Fischer and Jaikumar (1981) method using seed customers. Carranza (1997) presents other algorithms that tackle similar problems (v.g. school bus routing), but none of them consider the existing restrictions in a waste-collection problem.

2.2 Problem Constraints

Few authors consider more than one or two of the many restrictions involved in the collection of solid waste. Or and Curi (1993), analyze the case of Izmir, considering zones, boundaries and solid waste generation; road network, distances and durations; service and process capacities and cost data. Baetz et al.(1989), Baetz and Neebe (1994), Kulcar (1996) and Chang et al. (1996) analyze, using linear programming, location points for transfer plants and waste disposal and incineration sites.

The purpose of this sub-section is to define as completely and precisely as possible all the factors that may influence the economic design of the routes for waste collection:

- *Urban Patterns*

This has been possibly the most important aspect ignored by researchers in the analysis of the problem. With two exceptions, Stern and Moshe Dror (1979) and Eglese and Li (1992), its importance has not been considered or analyzed. The importance of square grids (v.g. Cerdà - Generalitat de Catalunya 1995) has been largely emphazised by many authors [1]. Since this type of grid is extended on all Latinamerica, a particular emphasis will be made on it.

- *Capacity and labour restrictions*

The design of urban waste collection routes requires an economical-technical decision to define the size of zones. Besides the capacity of trucks (which is mainly of 9,000 kg. in Latin America) there are labour restrictions. Employees work mainly overnight, in terms of 7 hours. There are two types of extra-hours, 50% extra-hours (the first two hours after normal term) and 100% extra-hours (the following extra-hours). Optimum sizes of routes must be determined, considering vehicle's and crew's costs.

- *Physical Restrictions*

Rivers, railways and avenues are hard restrictions for vehicle routing . These restrictions break the ideal continuum that would allow to configure routes in any way desired (possible optimum designs are generated with many contstraints). This is an important fact to consider when evaluating the possibility of automated designs and enforces the usefullness of manual designs.

- *Waste generation*

The analysis of waste generation is possibly one of the most complex problems to analyze in developing a design methodology. It is also one of the problems that has received less attention in the literature. Carranza (1997) mentions some of the main contributions done in this area, but the problem is far away from having been solved. He also proposes a method to determine values of waste generation in geographic zones. An extrapolation of waste generation values in censal radius (geographical zones smaller than a route used for population census) is done, taking into account quantities of houses per zone. Even though this work does not arrive to definitive conclusions, it is a step towards a complete analysis of the problem.

2.2.1 The Case of "Students-City"

A Latin-american city of 2.000.000 inhabitants has served to develop and test some aspects of the theory proposed in this paper. Figure 1 shows natural and man-made restrictions of this city, including highways, rivers and railroad.

Appendix B gives values of costs for operating the recollection service and table 3 shows ranges of optimal values for operative policies in the city. Figures 13, 14, 15

[1] Kostof (1991), Kostof (1992), Galantay (1975), Gallion (1986), Morris (1974), Chueca Goitia (1990), Lynch (1990) and García Espeche (1994) and enhance with different emphasis the importance of the square grid.

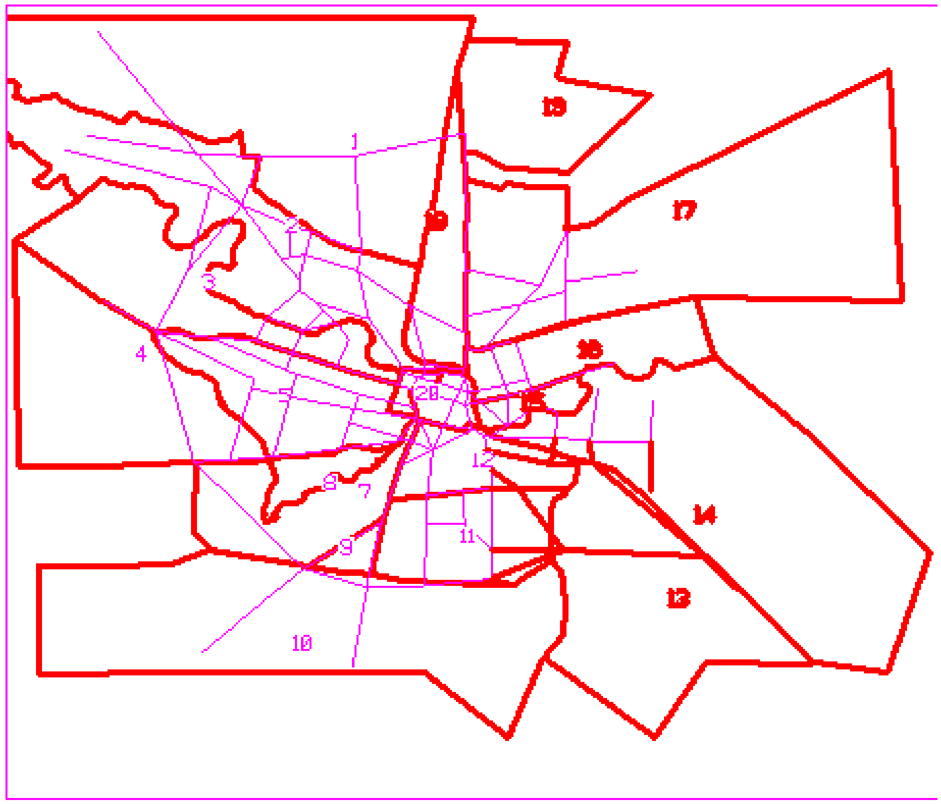


Fig. 1: Regions generated by natural and man-made restrictions in "Students-City"(numbers indicate regions)

and 18 show some regions of the city, including its grid. Equi-distance line contours are defined in figure 16.

The city is operated in the classical way of most latin-american cities, with crews of a driver and two loaders per truck, working each one on an area until it is completely served (employing an average of two trips per day) and collecting an average of 700 tons per day on a 6 days per week schedule.

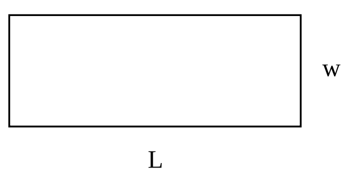
2.3 Continuous Approximations in the Design of Vehicle Routes

Newell (1973) gives some examples of this type of analysis. The objective is to "analyze some operations research type problems, which are basically finite-dimensional, by converting them (approximately) into problems involving continuous functions".

There is an extensive literature in different types of problems, all referred to the Traveling Salesman problem (TSP) and the Vehicle Routing Problem (VRP): Han and Daganzo (1986) consider different strategies to design zones for distribution of goods from a depot to many demand points without transhipments and with a limited amount of time. Daganzo (1984b) determines the relationship between expected distance in the TSP and the shape of a zone. Newell and Daganzo (1986a) analyze the optimal width of zones in a radial ring network. Robuste et al (1990) propose using simulated annealing for fine-tuning the solution obtained with the continuous approach strategy.

Only two articles will be analyzed, mainly because this work is related to square-grid networks. Daganzo (1984a) analyze the problem of distributing many goods from a depot to its influence area. He finds a formula to determine the optimum relationship between length and width for zones. It is simple to demonstrate that zones should be elongated towards the depot; this reduces access distances to zones. But since local travel distance is a function of width there is an optimum relationship $\beta = W/L$ between L and w, where:

$$\beta = W/L$$



With a rectangle oriented in its main direction towards the depot, Daganzo (1984b) demonstrate that the optimum width is $w^* = \sqrt{3/\delta}$ where the tour is done going through half of the points (on one half of the width) in one sense, and coming back through the other half, this is equivalent to $\beta * C = 6$, where

$w^* = \sqrt{3/\delta}$ = density of points to be served;

C = capacity of vehicle.

Optimum $\beta = w/L$ is obtained from minimizing average distance per point $\overline{d}$, which is determined from the following equation:

$$\overline{d} = (2/C)[\rho - L/2] + d^*, \quad \text{if } \overline{d} >= L/2 \tag{1}$$

$$= d^* \qquad \text{otherwise}$$

where

$\overline{d}$ = distance from depot to the center of the analyzed area

d^* = average local distance traveled per point

Zones orientation

A balance must be made in considering the orientation of zones with respect to a square grid and the orientation of isochrones with respect to the same grid. Daganzo and Newell (1984b) analize this problem. They study the distance between two points distributed in a Poisson process in a two-dimensional space (figure 2) in a not oriented grid.

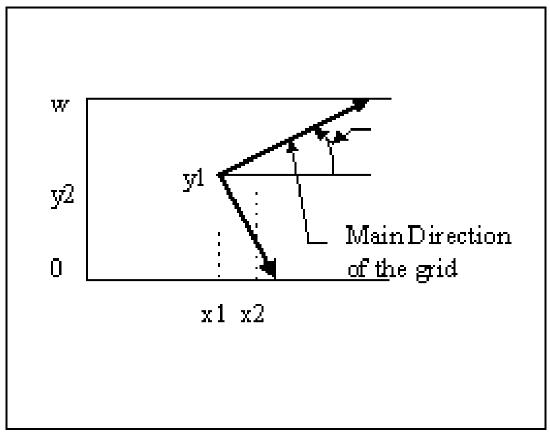


Fig. 2: Local distance savings according to zone orientation

It is deducted by symmetry of orientation towards the two axes, that there is a maximum for $\theta = 45^o$. As can be seen from the figure, θ is the angle defined by the one of the two directions of the grid and the main direction of the zone defined.

The other aspect to consider is line-haul distance per point. This distance per point varies with the orientation of isochrones with respect to the square grid. The angle ϕ between isochrones and the main direction of the square grid will depend on the minimum path between the zone and the depot, and will be influenced by any of the fast ways that may be utilized in this path. If there aren't fast ways near the zone analyzed the only way to travel between isochrones, will be along the direction of the grid. The difference in time between isochrones will be equal to the separation measured along the direction of the grid oriented more favourably (in figure 4, vertical direction).

Savings in line-haul in units of distance traveled in the local grid can be deduced (d'C in figure 3), obtaining the following formula in terms of ϕ and θ.

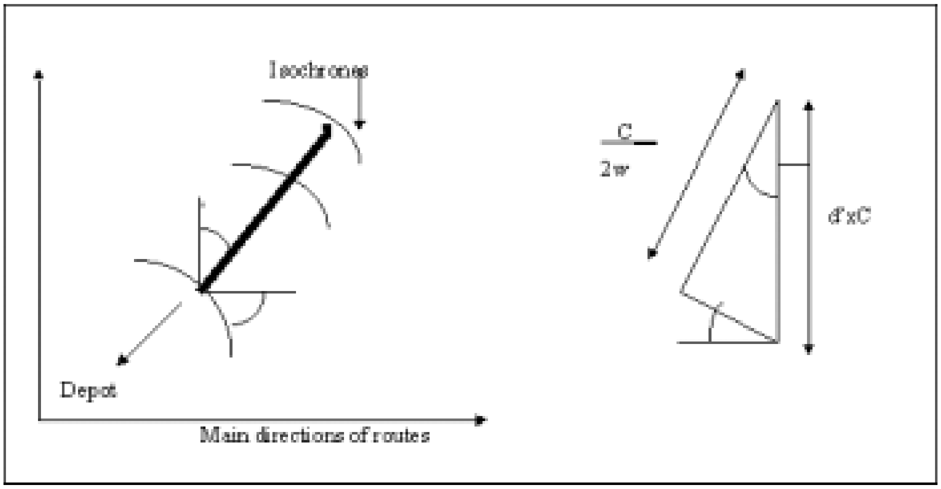


Fig. 3: Line haul distance saved according to grid orientation

The maximum value of this function will be for $\theta = \phi$ which corresponds to the geometric condition that the rectangles are oriented with their main side perpendicular to isochrones. The value to minimize is d-d'. For our case, in which we have a square grid, withouth fast roads (uniform speed in all the grid), equitravel time contours should be concentric squares oriented at 45^o to the grid lines with center at the source. Therefore $\phi = \pi/4$. There is a maximum for d and d' at $\theta = \pi/4$, and d-d' has a minimum at that value.

3 A Continuous Approximation Approach for the Chinese Postman Problem

Optimum relationships (width/length) for routes in square grid metrics, and oriented arcs succesively changing its orientation will be determined. The total length of a CPP tour is the sum of the segments of the network plus the distance generated by the minimum matching of odd-paired nodes. This formula may be applied either to directed or undirected graphs. An Euler tour may be constructed with the same distance in either directed or undirected graphs, if there are not unaccesible or not comunicated nodes[2]. Figure 4 shows a tour for a simple graph that can be traveled in a distance that differs in 4l from oriented to not oriented graphs. It's analysis leads

[2] Unaccesible or not comunicated nodes appear when a vertex has only ingoing or outgoing arcs.

us to a simple caracterization of the problem. According to Daganzo and Newell (1984b), the problem must be analyzed for different orientations of the grid towards equitravel time contours.

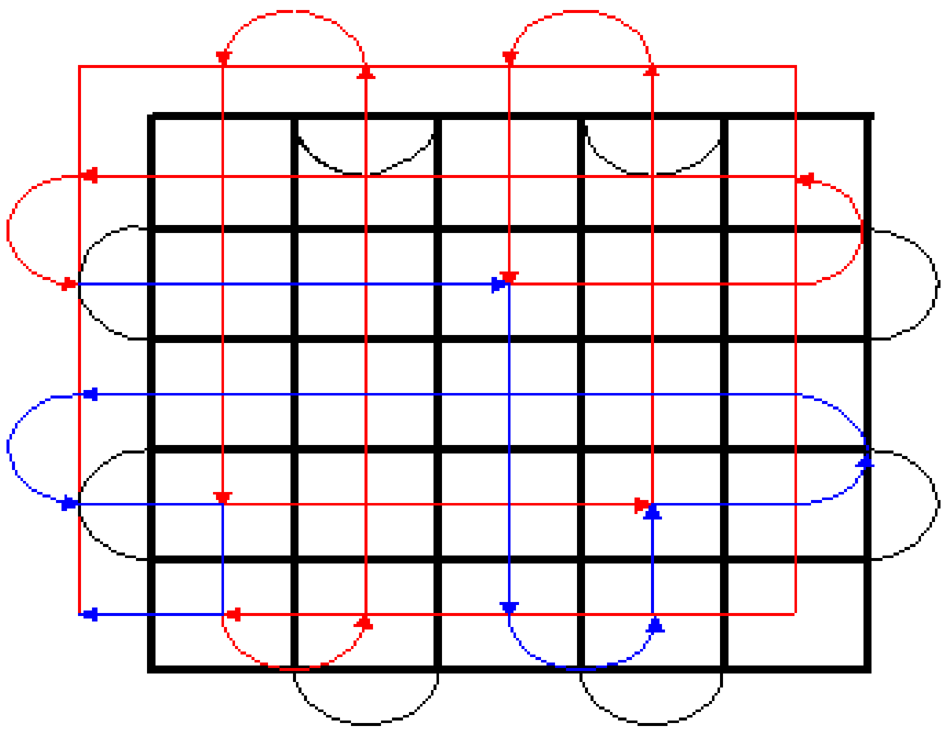


Fig. 4: CPP tour for a not oriented "Square grid" graph

3.1 The Total Distance Traveled for a CPP Tour in Graphs Oriented at 0^o with Respect to Equitravel Time Contours

Figure 5 presents a simplified graph. There is a compromise between the savings that can be obtained elongating a route towards the depot (make L larger) with the augmentation of odd nodes per unit perimeter (and it's consequent augmentation of local distance traveled) that ocurs because of the decrease of w.

Figures 6 and 7 show two of the cases that can ocur with respect to odd nodes:

1) An odd quantity of odd nodes in one side and an even quantity of odd nodes in the other side (Links in the figure make the union of odd nodes).

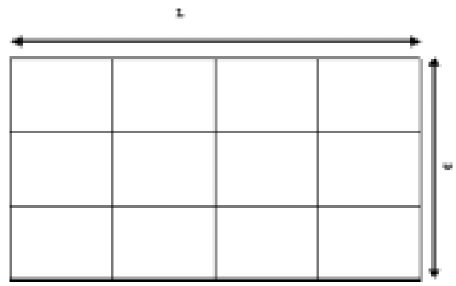


Fig. 5: Rectangular Area

2) Odd number of odd nodes in both sides

3) Even number of odd nodes in both sides

The total length of the tour will be determined according to the existence (or not) of evenness in the number of odd nodes:

$$d_1 = L * (w/l + 1) + w * (L/l + 1)$$
$$d_b = d_1 + (L/l - 1) * l + (w/l - 1) * l + p$$

where

d_r = sum of the lengths of the segments of the graph

d_b = total local distance traveled in the CPP (in the future base case)

l = length of a square

p=2*l if total number of odd nodes is even

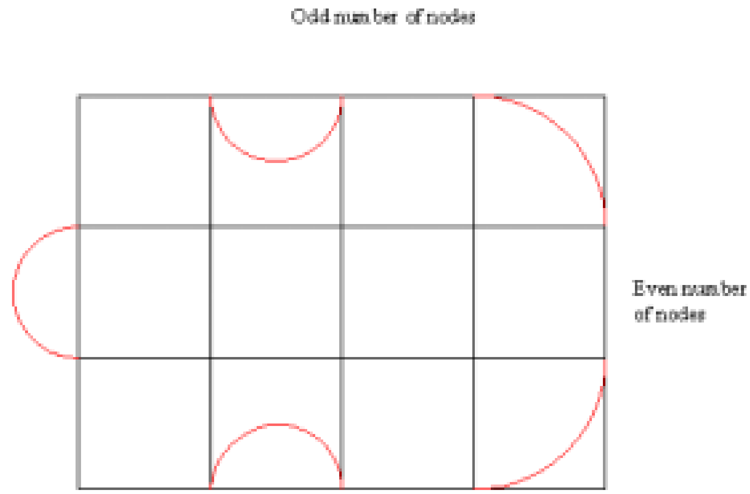
p=0 otherwise

Following the criterium proposed by Daganzo (1984a) (equations 1), we define the function to minimize:

$$f(L, w) = 2 * (L + w) - L$$

a function to minimize is determined:

$$f(\beta) = \sqrt{C/\delta}[2\sqrt{\beta} + 1/\sqrt{\beta}]$$



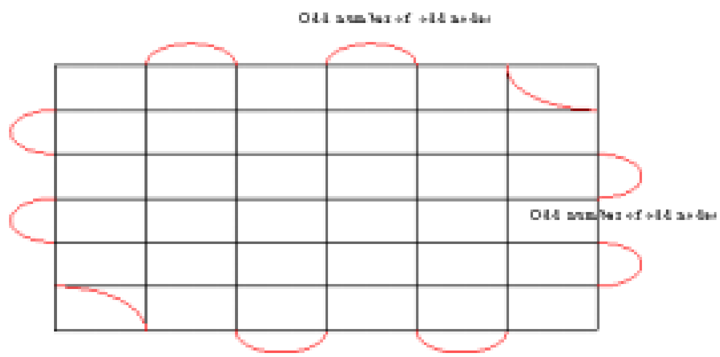


Fig. 6 - Graph with odd number of nodes on one side and even number of nodes on the other. Fig. 7 - Graph with odd number of nodes on both sides

doing $\sqrt{C/\delta} = K$

Figure 8 shows the function for $K = \sqrt{3}$.

and calculating the derivative

$$\frac{df(\beta)}{d\beta} = 3 * K * \frac{1}{\sqrt{\beta}} - \frac{3}{2} * K * \frac{1}{\beta^{3/2}}$$

it is obtained

$$\beta = \frac{1}{2}$$

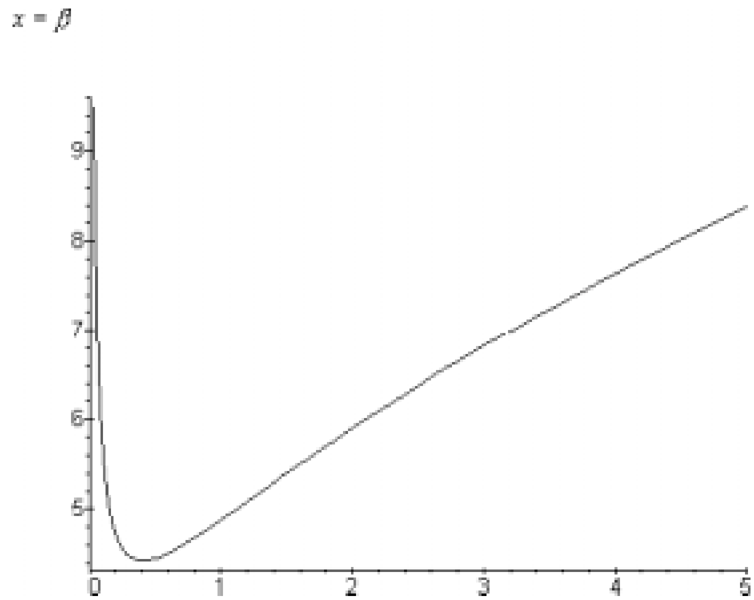


Fig. 8

which is independent of the value of K.

This first case shall be called "Base case" to distinguish it from further topologies of networks that will be mentioned.

3.1.1 The Existence of Inaccesible or Incomunicated Nodes

In general, any route that may be constructed over a network with alternative senses in both directions, there may be incomunicated or inaccesible nodes. It is evident that this situation should be avoided. Even though, we will deduct formulas to visualize optimum relationships of length-width. The extra-distance traveled, arises from comunicating nodes via links that do not belong to the route analyzed, but that exist in the periphery of it. One case will be analyzed, and optimum $\beta = \frac{1}{2}$ values will be given for other similar cases.

* Inaccesible points over side L (Case 1)

Extra-distance to travel is:

$2 * L + 4 * l$.

Total distance to travel is:

$d_1 = d_b + 2 * L + 4 * l$

The function to minimize is:

$2 * (L + w) - L + 2 * L$

whose derivative equaled to 0 is:

$$-\frac{3}{2} * \frac{\sqrt{K}}{\beta^{3/2}} + \frac{\sqrt{K}}{\sqrt{\beta}} = 0$$
$$\beta_1 = 0$$

With the same methodology, we can solve the problem for other rectangles with inaccesible or incomunicated nodes[3].

* Inaccesible-Incomunicated points on side w (Case 2) $\beta_2 = \frac{1}{4}$

*Inaccesible-Incomunicated points situated in the extreme points of one of the diagonals of the grid (Case 3) $\beta_3 = \frac{3}{4}$

*Inaccesible-incomunicated nodes in the four extreme points of one of the rectangle (Case 4) $\beta_3 = \frac{3}{4}$

3.1.2 Other Cases

Other cases may be found. Carranza (1997) gives a detailed exposition of some of them. These cases refer to variations of the base case, and their optimums are determined with the methodology that has been exposed for base case and Case 1.The cases analyzed are:

*Base case with one diagonal: Defining the values a and b as the fraction of width and length of the coordinates of the diagonal over the same width and length, the optimum $\beta_3 = \frac{3}{4}$ varies between 0.6 and 0.7 for relationships of a/b =1, but the optimum value bet for a=0.1 and b=0.9 is 0.9, which indicates that for a large range of values a and b the optimum $\beta_3 = \frac{3}{4}$ is below 1.

*Base case with avenues: Two formulas for $\beta_3 = \frac{3}{4}$ can be found (being n the number of avenues existing in the grid). The first one, considering avenues along the largest side, L: $\beta_3 = \frac{3}{4}$

[3] Carranza (1997) gives a detailed explanation of the methodology.

The other one is considering the other possible case, with an avenue along the shortest side, w: $\beta_3 = \frac{3}{4}$

*Base case with senses in direction L alternatively changing by two streets (two possible cases are analyzed):

-Case 1 $\beta_3 = \frac{3}{4}$

-Case 2 $\beta_3 = \frac{3}{4}$

*Base case with different average length of squares (assuming lw=length of side w and lL=length of side L).

l_w/l_L	$\beta_3 = \frac{3}{4}$
0.5	1/6
2	4/3

3.1.3 The Analysis of Distances Traveled Depending on Optimum Values of β

The possibility that the same zone may have optimum length-width relationships in a wide range of values leads us to analyze the percentual variation of distances traveled with the same values.Figure 9 shows different distances traveled for the base-case.

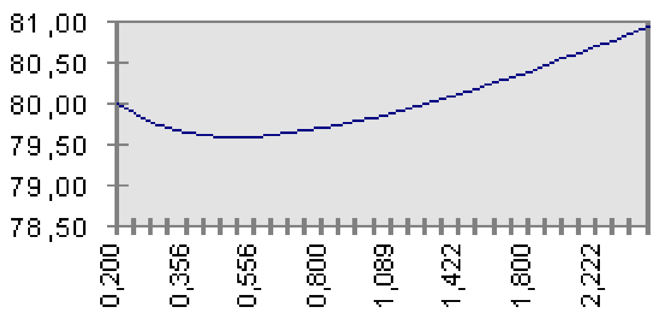


Fig. 9 - β vs. Distance for C=9 ton, δ=5 ton/km2 and ρ=20 km

The variation of a distance traveled in a range will give an idea of the grade of approximation of the measure. Doing a continous approach (e.g. approximating to the problem through its main variables), general information about waste generation in Latin America is extrapolated. The density of generation of waste, varies between a minimum of 1 ton/km^2 and a maximum of 20 ton/km^2 for a crowded commercial area. Considering trucks of 9 tons of capacity[4], the ratio C/δ (area to be served) will vary between 9 and 0.45 km^2.

Different tables (see appendix A) show values of distances traveled for different values of ρ and ratios C/δ, in the range previously defined. A value of l=0.1 km has been considered. Values of l=0.2 km give variations not larger than 1% of these values. Analyzing the percentual variation of all possible designs (given a particular case) with respect to the optimum for that case, can give an idea of the "goodness" of the different values of β. Table 1 summarizes these percentages for a particular case, among all the situations considered in Appendix A. For these particular values of ρ, δ and C/δ, the average difference of $\beta = 0.5$ (the diseconomy that should be expected to exist if we chose this value for the design of the route and every case considered ocurred with the same probability) with respect to optimum values for each case is 0.8%.

β	Base case	Case 1	Case 2	Case 3	Case 4	Average difference
0,17	1%	7%	0%	4%	0%	2,4%
0,23	0%	5%	0%	2%	0%	1,5%
0,50		2%	1%	0%	2%	0,8%
0,73	0%	0%	1%	0%	3%	0,9%
1,51	1%	0%	4%	1%	7%	2,5%

Table 1- Percentual variation of distances traveled for ρ=5 km, δ=10 ton/km^2, C/δ=0.9 km^2s

This table is representative of the practical results from the other tables. Taking into account the same analysis for the other cases presented in Appendix A, the average difference for β=0.5 and β=0.75 is always the minimal difference (among the 5 possibilities), and it's value is never bigger than 1,4%. Almost all cases except one are lower or equal to 1,1%. This considerations lead to assume that the optimum value for β in grids oriented at 90^o with equitravel time contours is between 0,5 and 0,75.

The following conclusions can be obtained from the tables in Appendix A:

1) For ratios $C/\delta \leq 0.45$ the formulas have errors of 4-6%. It would be the case of zones with a great density of inhabitants. The reduced dimension of this zones needs a detailed analysis.

2) The distance traveled is relatively insensitive to the variation in ρ (1-2%).

3) The base case (followed by case 1) presents the least relative percentual variation. This would lead us to generate in general zones with β=1/2.

[4] This type of trucks is the most widely used in Latin-America for their maniobrability.

3.2 The Total Distance Traveled for a CPP Tour in Graphs Oriented at 45^o with Respect to Equitravel Time Contours

When orienting zones at 45^o two cases may ocur: On one side, a zone may not have inaccesible or incomunicated nodes (Figure 10-Case A and figure 11). On the other, there may be multiple incomunicated or inaccesible nodes (see figure 12-Case B and figure 10).

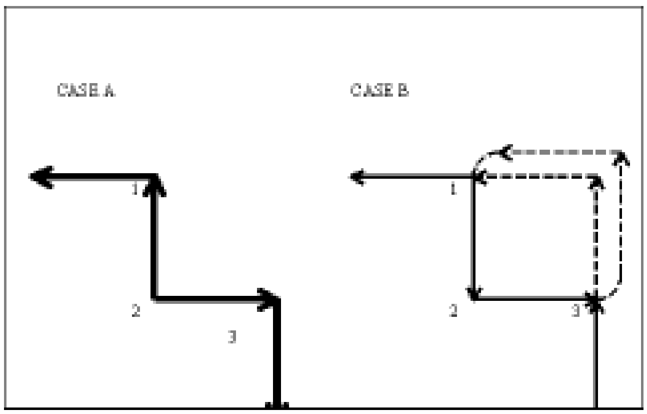


Fig. 10 - Possible cases in extreme nodes in zones oriented at 45^0 with the grid.

Zones oriented at 45^o *- Case A*

As with zones oriented to 90^o with the grid, we estimate the distance traveled within the grid. The distance of the segments (d_1) is:

$$d_1 = 2 * w * \cos(\frac{\pi}{4}) * (\frac{L}{l} * \cos(\frac{\pi}{4}) + 1) + 2 * L * \sin\frac{\pi}{4}(\frac{w}{l} * \cos(\frac{\pi}{4} + 1))$$

The function to minimize will therefore be:

$$f(\beta) = K * [\sin(\frac{\pi}{4}) * K * (2 * \sqrt{\beta * \frac{1}{\beta} + 2 * \sqrt{\beta}}) - \sqrt{\beta} * \frac{1}{\beta}]$$

doing the first derivative and equaling to 0:

$$\frac{\partial f(\beta)}{\partial \beta} = -0.205 * \frac{1}{\beta^{3/2}} + \frac{1}{\sqrt{\beta}}$$

$$\beta = \frac{1}{5}$$

which is a relatively low value, what could be supposed in view of the relevance that line-haul distance should have in this case, since there is no extra-distance.

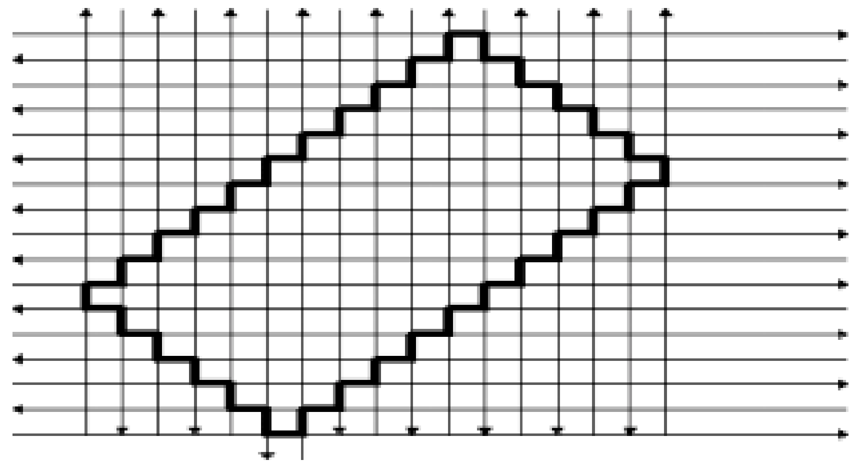


Fig. 11 - Case A -Zones at 45^o with no extra-distance for nodes in its perimeter.

Zones oriented at 45^o *- Case B*

The extra-distance that arouses from incomunicated or inaccesible nodes in the boundary of the zone is added to the distance of the segments (d_1),. This value is:[5].

$$d_b = w * \sin(\tfrac{\pi}{4}) * 2 + L * \sin(\tfrac{\pi}{4}) * 2 + 4 * l$$

which gives the following equation that has to be minimized:

$$f(\beta) = K * [\sin(\tfrac{\pi}{4}) * [\sqrt{\beta} * K * 4 + \sqrt{\beta} * \tfrac{1}{\beta} * 4] - \sqrt{\beta} * \tfrac{1}{\sqrt{\beta}}]$$

searching for the minimum:

$$\frac{\partial f(\beta)}{\partial \beta} = \frac{1 * 4 * \sin(\frac{\pi}{4})}{2 * \sqrt{\beta}} - \frac{1 * (4 * \sin(\frac{\pi}{4}) - 1)}{2 * \beta^{3/2}} = 0$$

$$\beta = 0.65$$

[5] The designer should be aware that this formula was determined for squares with equal sides. In the case that there different sides, the formula should be modified accordingly. The value of β may be 0.41 for l_1 =0.1 and l_2=0.2

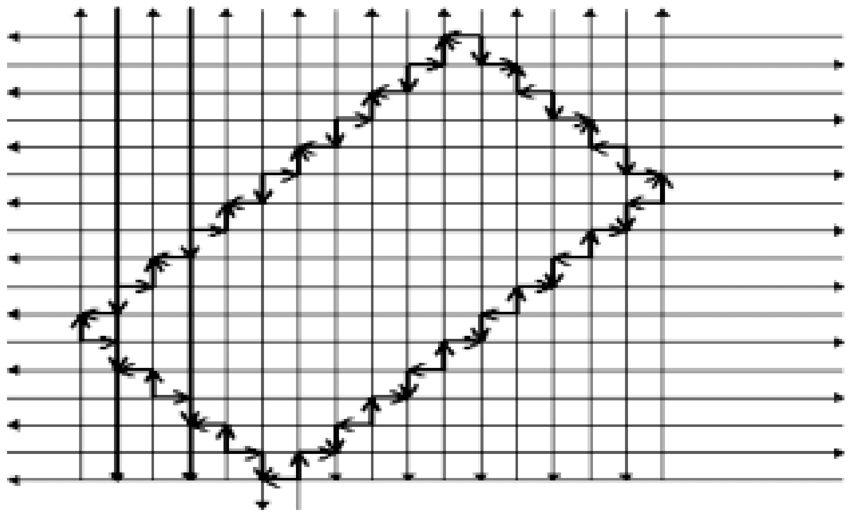


Fig. 12 - Zones at 45^o with extra distance because of nodes in its perimeter.

3.3 The Influence of the Orientation of Zones in the Total Distance Traveled

Formulas for two possible orientations of zones have been developed: at 45^o and at 0^o. Intemedium possibilities[6]are rare and it doesn't seem practical for a semi-maunal method to analyze more possibilities. If there were no fast roads and the network were a uniform speed rectangular grid everywhere, the equitravel time contours would be concentric squares oriented at 45^o to the grid lines with center at the source ($\phi = \pi/4$)[7]. It has been mentioned in point IV that there is a maximum saving of line-haul distance when the grid is oriented at 45^o to isochrones (equi-travel time contours). But the complexity of urban patterns will not always make possible that this occurs. Examples from "Students city" illustrate some situations that that may be found in cities with square grid patterns. Figure 13 shows the pattern of a zone named 15: The right half of this zone is elongated perpendicularly to isochrones. The orientation of the grid coincides with the perpendicular to isochrones.

Figure 14 shows a similar situation in zone 18. Even though routes may be elongated perpendicularly to isochrones, the width of the natural does not permit this situation. Figure 15 shows the typical pattern of 45 orientation.

The minimal distance achievable in this context is that traveled in networks of the

[6] Newell and Daganzo(1986) have determined maximums and minimums must be between these two values. See section II-3.
[7] ibidem

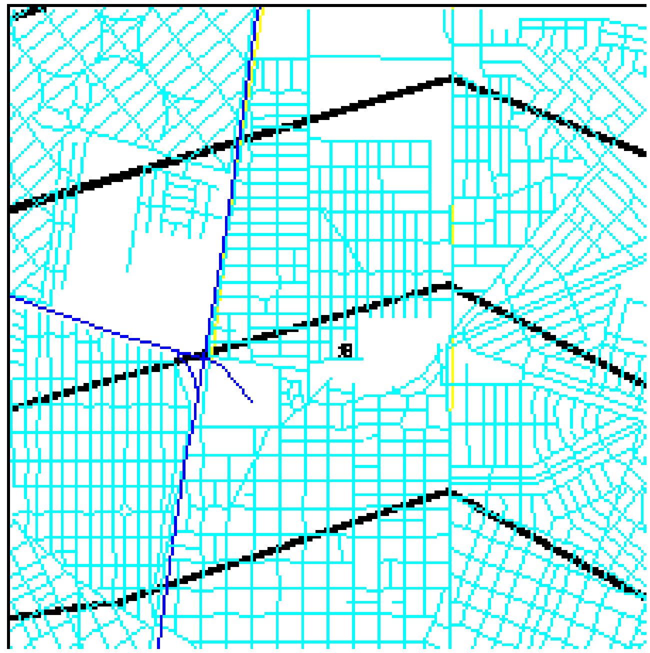


Fig. 13 - Zones with 0^o orientation.

type of Case A in this section: It does not have any extra-distance to be traveled in its perimeter (since there are no odd nodes in it). The savings that may be achieved in this context with respect to the Base Case defined in section III-1, according to the extreme values for the parameters C/δ and C/δ determined in section III-1-c can be seen in table 2.

C/δ	Area=C/δ	Percentual variation with respect to Case A
5	0.45	7
5	1.8	5.5
10	0.45	2.3
10	1.8	2.5
20	0.45	1.4
20	1.8	1.8

Table 2-Comparison of local distance and Line-haul savings for 0 and 45^o orientations[8]

[8] A deduction of the values showed in this table, can be seen in Carranza (1997)

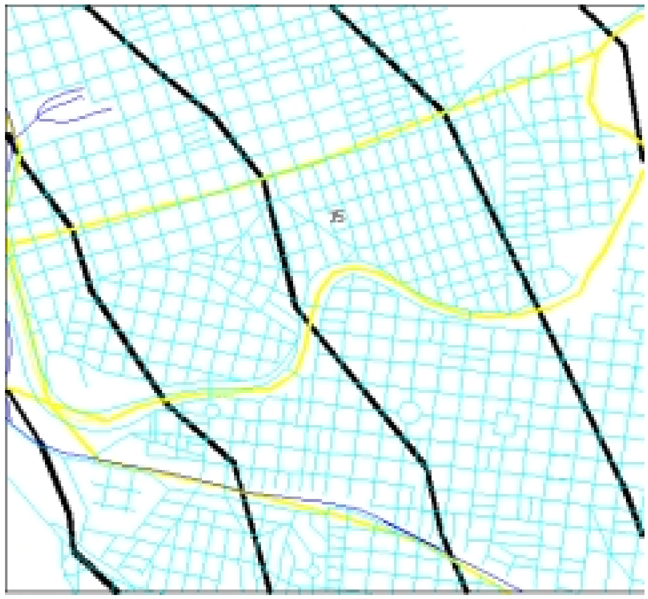


Fig. 14 - Zone18-Orientation at 0^o.

4 A Procedure to Design Operation Schemes

Formulas have been developed to determine optimal values of slenderness in routes, when the grid is a square grid, with senses of streets changing alternatively every one street and without odd nodes inside the route, which will be defined in section V as networks with high distance relationships. Some steps have to be followed for the design of routes, which are suscintly cited.

1 - Determination of equi-travel time contours

The determination of equi-travel time contours is the first step towards the design of a vehicle routing scheme for waste recollection. With the determination of rapid ways and a simple algorithm (e.g. Dantzig-1962), equitravel time contours are easily drawn in a Geographic Information System. Figure 16 shows equi-travel distance contours from the operative base, with distances measured in meters. Isochrones and Equi-distance contours coincide because the travel speed is supposed to be constant all over the network, assumption that is made over the night hours.

2 - Determination of size of zones

For the determination of optimal sizes for routes a balance should be made between crew costs and vehicle costs. Appendix B describes the equations that lead to

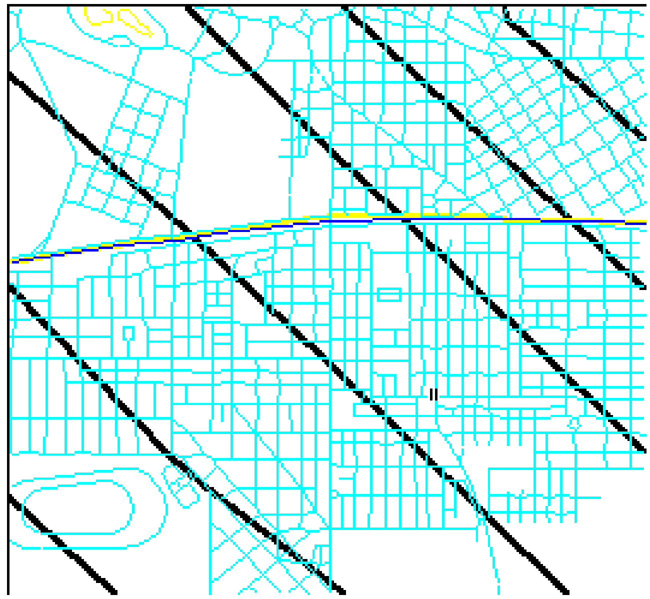


Fig. 15 - Typical pattern at 45^o.

the determination of Table 3. In this table the optimal operating policy is determined according to minimum costs. Two possible variations are analyzed for productivity, constant productivity (block 1) and variable productivity (block 2).

The operation consists in a trip from the operative base to the collection area, returning to the disposal site (the operative base is on the way to the disposal site for any area) and returning to the collection area if necessary. Workers work in "normal hours" (the first 7 hours), 50% extra hours (the two following hours) and 100% extra-hours (the following hours).

Three main operative policies are considered: Generating routes with one trip (COS 1 V) using only normal hours, generating routes using 50% extra-hours[9] (COS(50)), and generating routes using 100% extra-hours(COS(100)), that is, a second trip untill the capacity of the truck is complete. A fourth policy would consist in using 13 ton trucks and one trip. Shaded cells present the ranges of values (in distances from the depot) in which each policy is optimal.

According to the operating policy that is selected, the routes will have different

[9] Two trips to the disposal site will in general be used, depending on the distance of the point of collection from the disposal site. The crew will work untill it reaches the limit of the 50% extra hours.

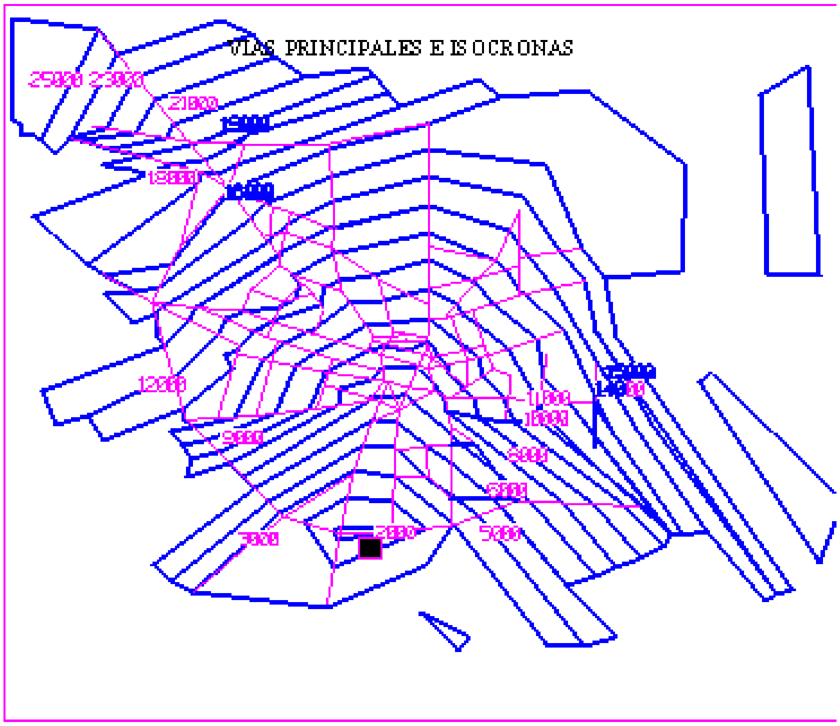


Fig. 16 - Equi-distance contours in "Students-City".

sizes: The route served in a second trip working in COS(50) will have a smaller size than the route served in a first trip (if the areas defined have the same waste density), which will complete the capacity of a 9 ton truck, if that were the type of truck used.

The generation of a table of the type presented in the appendix may be adequated to any case, and in some cases may generate important economies[10]. With the value of adequate sizes, zones may be generated within the regions defined in figure 1, assuring that a sufficient number of zones is generated, taking into account that their sizes should be near the optimum size determined in table 3.

3 - Design of zones

The first decision to make is the orientation of the zone with respect to isochrones. Table 4-a shows the different values of β for the different situations that may occur. As has been analyzed in section III-1, there are ranges of values of β in which total

[10] Carranza (1994) shows some of the economies that may be achieved with a proper definition of the size of zones.

X (distance from the depot)	*(1) Variable ratio of Prod.* Normal hours=2 t/h 50% Extra-hours=1.5 t/h 100% extra-hour=1.3t/h		*(2) Constant Ratio of Prod.* 2 t/h		(3) 13 t. 1 Trip COS(13)
	COS(1 V)	COS(50)	COS(50)	COS(100)	
0-8	10,1-11,9	7,9-11,5	5,9-6,9	7,0-9,3	7,7-9,3
9-15	12,1-13,4	12,1-17,8	7,3-10,9	9,6-11,4	9,6-10,9
16-25	13,6-15,6	19,2-59,2	11,8-36,7	11,7-14,4	11,1-13,1

Table 3

distances remain within a reasonable percentual variation. But this values were defined for graphs oriented at 0^o with respect to equi-travel time contours. Section III-2 analyzed the total distance traveled for a CPP tour in graphs oriented at 45^o with respect to equitravel time contours. Table 2 showed that the percentual variation in total distance traveled with respect to the optimum, may vary in a range between 1.8-7%[11]. This aspect should then be considered with more care than the ideal slenderness deduced in section III-1 (see table 1).

Carefull attention should be paid to these details. The selection of one or other orientation $(0^o - 45^o)$ is a matter not only of theoretical optimality, but of practical implementation. The existence of inaccesible or not comunicated points in the perimeter of zones is also important and should be considered in the design of the zones.

Orientation (^{o})	Case	β
45	A	0,2
45	B	0,65
0	BASE	0,5-0.75

Table 4-a Optimal values of β

The grid analyzed is one with a high distance relationship (see section V), with alternative senses in its streets. Figures 17 and 18 show different stages of the procedure that must be followed for the design of a zone. An auxiliar grid is used to define zones which do not exceed the dimensions determined in the second step of this procedure. In the case analyzed (region 8 of figure 1), routes have been built in an iterative way, giving a lower slenderness factor β to routes whose limits do not coincide with the perimeter of the region (routes 2, 3 and 5). The other routes have values that tend to be equal to 0.5, according to the appearence of odd nodes in the perimeter. As a general criteria, the designer must have in mind that there is a continuum for the optimum values of slenderness factors β which varies between

[11] The value varies according to the size of the zone and the line-haul distance.

0.2 for routes in whose perimeter there are not odd nodes (ideally case A for routes oriented at 45^o with isochrones, Section III-2), and 0.5 for routes with a "reasonable minimum" of odd nodes in its perimeter (Base Case in Section III-1).

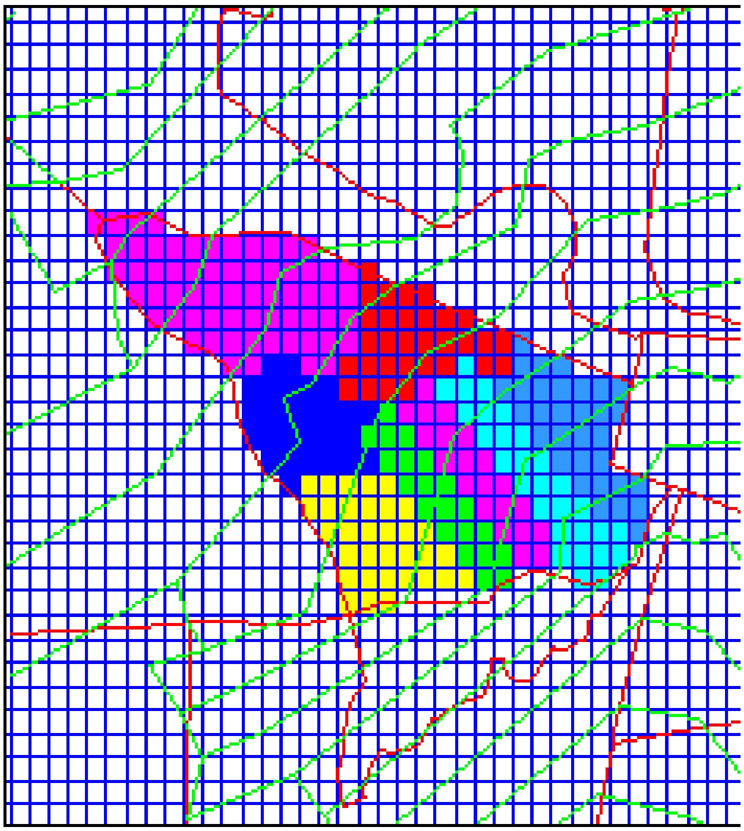


Fig. 17 - Auxiliar grid to construct a zone.

The dimension of each route or zone is determined according to its distance from operative base. Since the distance is of 9 km, and from table 3, it can be seen that designing routes for 1 trip is the optimum. Since the tonnage generated in the area is of 81 tons, 9 routes of approximately 9 tons each, are generated. The area is determined taking into account the waste's density (figure 17). Table 4-b summarizes the characteristics of the different routes showed in figure 18. The difference in sizes arises because of differences in density, an aspect analyzed by Carranza (1997).

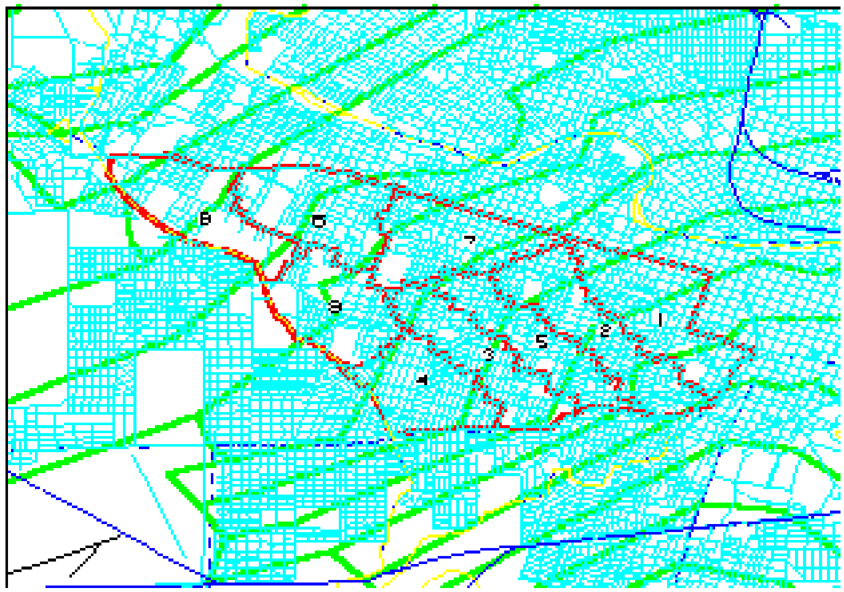


Fig. 18 - Definitive configuration of the zone.

Zone or route	β	Area km^2
1	0,42	2
2	0,2	1,94
3	0,2	1,62
4	0,5	1,52
5	0,2	1,52
6	0,6.	1,85
7	0,55	1,76
8	0,54	1,74
9	0,68	1,66

Table 4-b - Region's 5 Characteristics

4 - General procedure

The procedure is iterative in nature (it's authomatization, at least for this type of networks, seems far away from the actual state of the art in computer science) and is summarized in figure 19. As has been stated, carefull attention should be paid to details, taking into account the particular characteristics of the regions where routes are being designed.

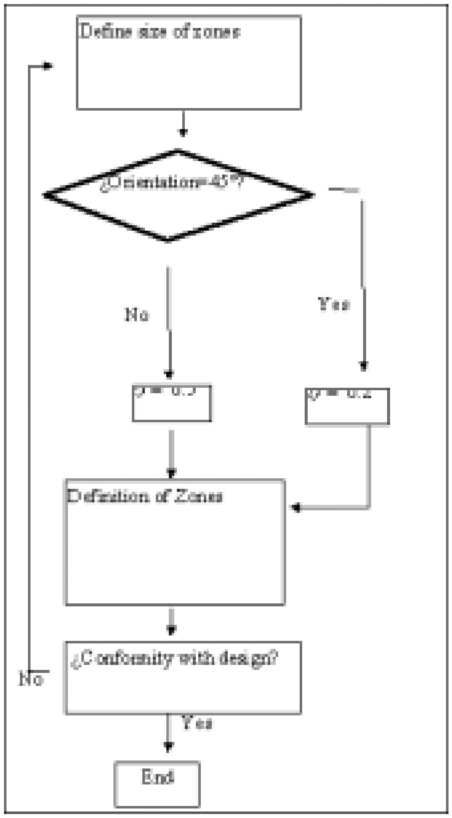


Fig. 19 - General criteria for the design of zones.

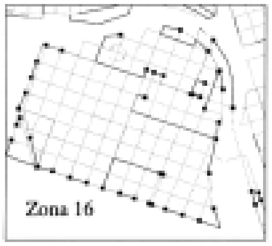
5 The Analysis of Networks with Low Values of Distance Relationships

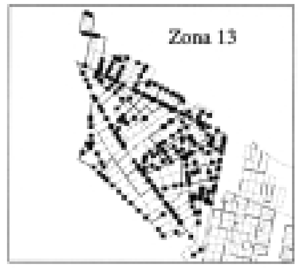
Eglese and Li (1992) propose a classification of grids according to the relationship of length of the arcs of the network with respect to total length to travel on the grid in the CPP, which will be further defined as distance relationship. According to this classification, square-grids would be in the order of 0.8-0.9 for the extradistance relationship, and rural networks would have a lower value (0.6-0.7).

When analyzing square grids for a city as the one that has been analyzed, some considerations have to be made. The two main hypothesis formulated in section three were that there would be alternative senses every one street, and that there existed no interior odd nodes. Considering this, the formulas presented in section three are also applicable to not oriented networks even with interior odd nodes. But oriented graphs with interior odd nodes remain unexplored.

5.1 An Experiment to Analyze Networks with Interior Odd Nodes

Carranza (1997) presents an experiment that is relevant in the development of a theory that explains the generation of extradistances in every type of network. Considering 15 different types of topological configurations, senses are asigned in a somehow random way (defining them to be different from the classical alternative orientation analyzed in section III-1). Figure 20 presents some of the configurations analyzed. Odd nodes are represented in black points. Table 5 presents values for distance relationships.





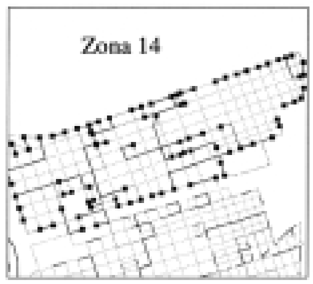


Fig. 20 - Configurations analyzed.

The solution to the CPP for all the zones was obtained with the algorithm of Lin and Zhao (1989). Zones are characterized by their extra-distance relationship and by quantities and densities of odd nodes. Several correlations where analyzed, some of which are summarized in Table 6.

This leads us to get some conclusions from the experiment. Intuitively it can be said that total quantity of odd nodes generate total extradistance, which can be seen in row 5 of table 6. The difference between total nodes and critical nodes[12] explain as well (with a similar value of R^2) the extradistance (row 7 of table 6).

Results from the experiment suggest two conclusions:

1) Interior odd nodes have a greater influence in total extradistance than perimetral odd nodes.

2) Supercritical node density of odd nodes gives a better explanation of the variation of travel distances than absolute density of odd nodes.

[12] Critical nodes are defined as the quantity of odd nodes that must exist in the perimeter of the zone if the zone is rectangular and has the square-grid pattern.

Zone	Distance Relationship
7	0,67
8	0,60
9	0,54
10	0,51
Zone	Dist.Relation.

11	0,55
12	0,57
13	0,40
14	0,65
15	0,61
Zone	Dist. Relation.

16	0,71
17	0,57
18	0,60
20	0,49
21	0,58
Zone	Dist. Relation.

22	0,51
23	0,63

Table 5

F	Dependent variable	Independent variable	R^2	R	Fcr.	F	p<
1	Distance relationship	Interior odd nodes density	0,82	0,90	4.54	59	.0
2	Distance relationship	Interior nodes/ perimeter	0,72	0,85	4.54	34	$6*10^{-5}$
3	Distance relationship	(total nodes – critical nodes) /area	0,70	0,84	4.54	31	$9*10^{-4}$
4	Distance relationship	Total odd nodes/area	0,63	0,79	4.54	22	$4*10^{-3}$
5	Total length of tour – total length of segments	Total odd nodes	0,81	0,90	4.54	57	0
6	Distance relationship	Total odd nodes/area	0,63	0,79	4.54	22	$4*10^{-4}$
7	Total length of tour – total length of segments	Total nodes –Critical Nodes	0,79	0,89	4.75	48	$1*10^{-4}$

Table 6

5.2 Other Conclusions

The results of the experiment suggest that a thorough characterization is necessary for every possible network. Carranza (1997) proposes one, classifying grids according to ranges of distance relationships and dividing networks in two main groups (oriented or not oriented) and within them, considering type of grid (rural or square grid) and interior odd nodes density (table 7). This classification takes also into account Eglese and Li's considerations about the ranges within rural and square grid networks distance relationships may vary.

A final consideration can be made about a generic formula for all types of networks. For this purpose a formula developed by Daganzo (1984b) will be proposed. He determines the expected distance between two points when groups of them are distributed randomly with a normal distribution in zones of width w. Considering a square grid, the value would be:

Distance relationship range	Type of oriented grid	Interior odd nodes density	Type of not oriented grid	Interior odd nodes density
>0.90	Square grid without odd nodes	<5	Square grid	<40
<0.90 y >0.8	Square grid with interior odd nodes	>20 y >5	Square grid	>40 y <50
<0.80 y >0.7	Square grid	<30 y >20	Square grid or rural	>50 y <60
<0.70	Square grid	>30	Square grid o rural	>60

Table 7

$$2w + L + (\frac{w}{3} + \frac{L}{3}) * \frac{\Delta * A}{2}$$

This formula could be applied to calculate the minimum average distance that arises because of interior odd nodes. A modification is done to this formula (the main hypothesis that will be formulated). Daganzo supposed for the determination of the formula that randomness appeared in one direction (x), which gave an average distance of w/3. The other term of the equation $2w + L + (\frac{w}{3} + \frac{L}{3}) * \frac{\Delta * A}{2}$ is obtained by integration and because the positions at the side of the strip at which the points lie form locally a Poisson process with rate $2w + L + (\frac{w}{3} + \frac{L}{3}) * \frac{\Delta * A}{2}$ being $2w + L + (\frac{w}{3} + \frac{L}{3}) * \frac{\Delta * A}{2}$ the density of points in the zone. In this case a "double randomness" will be considered, that is, it will be supposed that the average extra-distance for interior odd nodes will be w/3 in both directions.

This supposition surges from the fact that distances of tours that link interior odd nodes are not constant. This could be visualized from the different solutions to the 15 networks analyzed. This fact may emerge from the imprevisibility of the sense of the streets, which are far from the ideal theoretical analyzed in section three.

Considering the base case, and supposing that extradistances in the perimeter are generated in an analogous mode as in section three (which is very reasonable), the following equation is deducted and minimized:

$$2w + L + (\frac{w}{3} + \frac{L}{3}) * \frac{\Delta * A}{2}$$

where:

A = Area of the analyzed zone

Δ = Interior density of odd nodes

$$\begin{gathered} P = \sqrt{A} \\ w = P * \sqrt{\beta} \\ L = \frac{P}{\sqrt{\beta}} \\ f(\beta) = \sqrt{\beta} * (2 + \frac{\Delta P^2}{6}) + \frac{1}{6\sqrt{\beta}} * (\Delta P^2 + 6) \end{gathered}$$

derivating:

$$\frac{1}{2} * \left(\frac{\frac{1}{6}\Delta P^2 + 1}{\sqrt{\beta}} \right) - \frac{1}{12} * \left(\frac{\Delta P^2 + 6}{\beta^{3/2}} \right) = 0$$

an optimum for β is obtained:

$$\beta = \frac{1}{6} * \left(\frac{\Delta P^2 + 6}{\frac{1}{6}\Delta P^2 + 2} \right)$$

This function tends to 0.5 when $\Delta P^2 \to 0$. This would indicate that the formula would include the base case analyzed in section three. In figure 21 optimum β has a low variation in a wide interval of values for ΔP^2 when β aproximates 1.

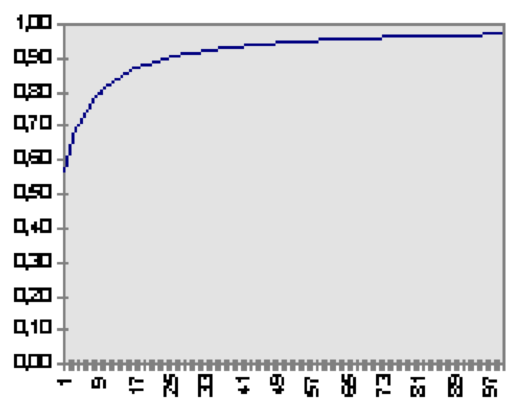


Fig. 21 - Value of the proposed function for P=2 and variable k.

6 Conclusions and Future Research

A methodology for the design of waste collection routes in networks of low distance relationship has been proposed. Optimal values for slenderness factors have been deduced, considering the two extreme different possible orientations of zones. The procedure has to be presented in a simple format to final users, people with a low degree of technical expertise. The process will probably be enhanced for use in automatized systems. Even though at the actual state of the art of graphic recognition by computers it does not seem to be applicable for this purpose, some contributions may be made. The development of DSS must be necessarily done for use in an interactive manner, including a GIS module.

Possibly the most important line of research that arises from the investigation done is that networks need a more exhaustive characterization than the one that has existed to the date. This classification would simplify the analysis and the application of solutions to any type of network, and therefore, make this method universally available. The classification and formulas that have been proposed in section six need more validation. This is a very interesting line of research.

The analysis of values of waste generation, succintly mentioned in section two, is a very important area of investigation. The work of Carranza (1997) gets in the subject but much more work is to be done. The economical implications of accurate forecasts of waste generation have a great impact in the final definition of a waste collection system, as Carranza (1994) demonstrated. The consideration of seasonal (and weekly) variations should be included in an analysis of this type.

References

Alvarez-Valdés,R., Benavent, E., Campos, V., Corberán, A., Mota, E., Tamarit, J.M., Vall, V. (1993) "Computer-aided design of routes for urban garbage collection", TOP-Vol. 1 (1) pags. 89-105.

Baetz B. W., Pas E., Neebe A., (1989) "Trash management:sizing and timing decisions for incineration and landfill facilities" Interfaces Vol. 19 Nro. 6.

Baetz B. W., Neebe A., (1994) "A Planning Model for the development of Waste Material Recycling Programmes" Journal of the Operational Research Society Vol. 45 Nro. 12 pags. 1374-1384.

Beltrami E., Bodin L.D., (1974)"Networks and vehicle routing for municipal waste collection" Networks Vol. 4 pags. 65-94.

Bodin L., Fagin G., Welebny R., Greenberg J., (1989) "The design of a computerized sanitation vehicle routing and scheduling system for the town of Oyster Bay, New York" Computers and Operations Research Vol. 16 Nro. 1 pags. 45-54.

Bodin, L., Kursh S., (1979) "A Detailed Description of a Computer System for the routing and Scheduling of Street Sweepers" Computers and Operations Research Vol. 6 pags. 181-198.

Carranza O., “Metodología de Cálculo de Costos de Sistemas de Recolección de Residuos”-ISIT-Universidad Nacional de Córdoba-Technical Report (1994).

Carranza O., “Diseño de Rutas de Recogida de Basuras mediante aproximaciones continuas”-Doctoral Dissertation-Universidad de Navarra (1997).

Clark G. and Wright J., (1964), “Scheduling of vehicles from a central depot to a number of delivery points”, Operations Research, Vol. 12, pags. 568-581.

Chang N., Yang Y.C.,Wang S.F., (1996) “Solid Waste management Systema Analysis with Noise control and Traffic Congestion Limitations“ Journal of Environmental Engineering Vol. 122 Nro. 2 pags. 122-131.

Chueca Goitia, Fernando Breve Historia del Urbanismo- Alianza Editorial (1990).

Daganzo C., (1984a) “The Distance Traveled to visit N points with a Maximum of C Stops per vehicle: An Analytic Model and an Application” Transportation Science Vol. 18 B Nro. 4 pags. 331-350.

Daganzo C., (1984b) “The Length of Tours in Zones of Different Shapes” Transportation Research B Vol. 18 B Nro. 2 pags. 135-145.

Daganzo C., Newell G., (1986a) “Design of Multiple Vehicle Delivery Tours I- A Ring Radial Network” Transportation Research B Vol. 20 B Nro. 5 pags. 345-363.

Daganzo C.,Newell G., (1986b) “Design of Multiple Vehicle Delivery Tours II- Other Metrics” Transportation Research B Vol. 20 B Nro. 5 pags. 365-376.

Eglese R.W., Li L.Y.O., (1992)“Efficient Routeing for Winter Gritting“ Journal of the Operational Research Society Vol. 43 Nro. 11 pags. 1031-1034.

Evans S.R.,Norback J. P., (1984)“An Heuristic Method for Solving Time-Sensitive Routeing Problems“ Journal of the Operational Research Society Vol. 35 Nro. 5 pags. 407-414.

Evans James R., Minieka Edward (1992), Optimization Algorithms for Network and Graphs-Marcel Dekker Inc. Eds.

Fisher M., Jaikumar R., (1981)“A Generalized Assignment Heuristic for Vehicle Routing“ Networks Vol. 11 pags. 109-124.

Generalitat de Catalunya, (1995)Cerdà, Ciudad y Territorio-Una Visión del Futuro.

Galantay, E, Nuevas Ciudades: De la Antigüedad a nuestros días Ed. Gustavo Gilli-(1975).

Gallion A., Eisner S., The Urban Pattern: City Planning and Design Van Nostrand Reinhold (1986).

García Espeche A., Ciudades, del Globo al Satélite Centre de Cultura Contemporanea de Barcelona (1994).

Gillet B., Miller L., (1974)"A Heuristic for the Vehicle-Dispatch Problem" Operations Research Vol. 22 pags. 340-349.

Glover, Fred (1967), "Finding an Optimal Edge Covering Tour of a Connected Graph" ORC-13, Operations Research Center-University of California at Berkeley.

Kostof-Spiro, The City Shaped-Urban Patterns and Meanings through History Thames and Hudson Ltd. (1991).

Kostof-Spiro, The City Assembled Thames and Hudson Ltd (1992).

Han A., Daganzo C., (1986)"Distributing Nonstorable Items without Transshipments" Transportation Research Record B Vol. 1061 pags. 32-41.

Larson R., Odoni A., (1980) Urban Operations Research -Prentice-Hall.

Lin Y., Zhao Y. (1988) "A new Algorithm for the Directed Chinese Postman Problem" Computers and Operations Research, Vol. 15 nro. 6 pags. 577-584.

Lynch K., City Sense and City Design MIT Press (1990).

Mei-Ko, Kwan (1962) " Graphic Programming using Odd or Even Points"Chinese Mathematics Vol. 1 pags. 273-277.

Morris A.E.J. Historia de la Forma Urbana (1974).

Murty, K. G., (1967) "The Symmetric Assignment Problem" ORC 67-12 University of California at Berkeley.

Newell G., (1973) "Scheduling, Location, Transportation, and Continuum Mechanics: Some simple approximations to Optimization Problems" Slam Journal of Applied Mathematics Vol. 25 Nro. 3 pags. 346-360.

Or I., Curi K., (1993)"Improving the efficiency of the solid Waste collection System in Izmir, Turkey, through Mathematical Programming" Waste Management & Research Vol. 11 pags. 297-311.

Robusté, F., Daganzo C., Souleyrette II R., (1990) "Implementing Vehicle Routing Models" Transportation Research B Vol. 20 B Nro. 4 pags. 263-286.

Stern H., Dror M., (1979) "Rounting Electric Meter Readers" Computers and Operations Research Vol. 6 pags. 209-223.

Stricker R., (1969) "Public Sector Vehicle Routing: The Chinese Postman Problem" Ms Sc Dissertation-M.I.T.

Appendix A

Tables 1 to 6 show distances traveled in square grids, using different values of β. Tables 7 to 12 show percentual veriations of distance of different velues of slenderness with respect to the optimum (minimum) distance for the particular case analyzed.

$\rho = 5$
$\delta = 20$
$C/\delta = 0.45$

	base case	case 1	case 2	case 3	A
β					
0.25	20.79	22.33	25.38	22.5667	0.08
0.50	21.17	22.32	26.52	23.4737	0.10
0.75	21.49	22.53	27.47	24.1913	0.11
1.50	22.37	23.19	30.12	26.0727	0.14
B	-0.06	0.4	0.14	0.16	

Table 1 - Distance traveled for $\rho = 5$, $\delta = 20$, $C/\delta = 0.45$.

$\rho = 10$
$\delta = 20$
$C/\delta = 0.45$

	base case	case 1	case 2	case 3	A
β					
0,25	30.79	32.33	35.38	32.5667	0.5
0,50	31.17	32.32	36.52	33.4737	0.07
0,75	31.49	32.53	37.47	34.1913	0.08
1,50	32.37	33.19	40.12	36.0727	0.10
B	-0.04	0.03	0.10	0.11	

Table 2 - Distance traveled for $\rho = 10$, $\delta = 20$, $C/\delta = 0.45$.

$\rho = 5$
$\delta = 10$
$C / \delta = 0.9$

	Base case	case 1	case 2	Case 3	A
β					
0.25	30.48	33.26	36.45	32.2333	0.05
0.50	30.65	32.74	36.94	32.9474	0.07
0.75	30.88	32.65	37.65	33.5826	0.08
1.50	31.65	32.99	39.94	35.3455	0.10
B	-0.03	-0.01	0.08	0.10	

Table 3 - Distance traveled for $\rho = 5$, $\delta = 10$, $C/\delta = 0.9$.

$\rho = 10$
$\delta = 10$
$C / \delta = 0.9$

	base case	case 1	Case 2	case 3	A
β					
0,25	40.48	43.26	46.45	42.2333	0.04
0,50	40.65	42.74	46.94	42.9474	0.05
0,75	40.88	42.65	47.65	43.5826	0.06
1,50	41.65	42.99	49.94	45.3455	0.08
B	-0.02	-0.01	0.06	0.07	

Table 4 - Distance traveled for $\rho = 10$, $\delta = 10$, $C/\delta = 0.9$.

$\rho = 5$
$\delta = 5$
$C/\delta = 1.8$

	base case	case 1	Case 2	case 3	A
β					
0.25	49.87	55.11	58.61	51.5667	0.03
0.50	49.59	53.58	57.78	51.8947	0.04
0.75	49.67	52.90	58.00	52.3652	0.05
1.50	50.19	52.58	59.57	53.8909	0.07
B	-0,01	-0.05	0.03	0.05	

Table 5 - Distance traveled for $\rho = 5$, $\delta = 5$, $C/\delta = 1.8$.

$\rho = 10$
$\delta = 5$
$C/\delta = 1.8$

	base case	case 1	Case 2	case 3	A
β					
0.25	59.87	65.11	68.61	61.5667	0.03
0.50	59.59	63.58	67.78	61.8947	0.04
0.75	59.67	62.90	68.00	62.3652	0.04
1.50	60.19	62.58	69.57	63.8909	0.06
B	-0.01	-0.04	0.03	0.04	

Table 6 - Distance traveled for $\rho = 10$, $\delta = 5$, $C/\delta = 1.8$.

β	Base case	case 1	case 2	case 3	case 4	Difference
0,17	1%	10%	0%	6%	0%	3,4%
0,23	1%	7%	0%	3%	0%	2,2%
0,50	0%	2%	1%	0%	2%	1,1%
0,73	0%	1%	2%	0%	4%	1,4%
1,51	1%	0%	5%	1%	9%	3,5%

Table 7 - Percentual variations of distances traveled for $\rho = 5$, $\delta = 20$, $C/\delta = 0.45$.

β	Base case	case 1	case 2	case 3	case 4	Difference
0,17	1%	7%	0%	4%	0%	2,4%
0,23	0%	5%	0%	2%	0%	1,5%
0,50	0%	2%	1%	0%	2%	0,8%
0,73	0%	1%	1%	0%	3%	1,0%
1,51	1%	0%	4%	1%	7%	2,4%

Table 8 - Percentual variations of distances traveled for $\rho = 10$, $\delta = 20$, $C/\delta = 0.45$.

β	Base case	case 1	case 2	case 3	case 4	Difference
0,17	1%	7%	0%	4%	0%	2,4%
0,23	0%	5%	0%	2%	0%	1,5%
0,50		2%	1%	0%	2%	0,8%
0,73	0%	0%	1%	0%	3%	0,9%
1,51	1%	0%	4%	1%	7%	2,5%

Table 9 - Percentual variations of distances traveled for $\rho = 5$, $\delta = 10$, $C/\delta = 0.9$.

β	Base case	case 1	case 2	case 3	case 4	Difference
0,18	1%	7%	0%	4%	0%	2,3%
0,28	0%	4%	0%	2%	0%	1,3%
0,54	0%	1%	1%	0%	2%	0,8%
0,71	0%	1%	1%	0%	3%	1,0%
1,47	1%	0%	4%	1%	7%	2,5%

Table 10 - Percentual variations of distances traveled for $\rho = 10$, $\delta = 10$, $C/\delta = 0.9$.

β	Base case	case 1	case 2	case 3	case 4	Difference
0,17	1%	8%	0%	5%	0%	2,9%
0,23	1%	6%	0%	3%	0%	1,8%
0,50		2%	1%	0%	2%	1,0%
0,73	0%	0%	2%	0%	4%	1,1%
1,51	1%	0%	5%	1%	8%	3,0%

Table 11 - Percentual variations of distances traveled for $\rho = 5$, $\delta = 5$, $C/\delta = 0.9$.

β	Base case	case 1	case 2	case 3	case 4	Difference
0,18	1%	7%	0%	4%	0%	2,4%
0,28	0%	5%	0%	2%	0%	1,5%
0,54	0%	2%	1%	0%	2%	0,8%
0,71	0%	0%	1%	0%	3%	0,9%
1,47	1%	0%	4%	1%	7%	2,5%

Table 12 - Percentual variations of distances traveled for $\rho = 10$, $\delta = 5$, $C/\delta = 0.9$.

Appendix B

This appendix determines values for different operative policies in "Students-City", depending on the distance from operative base. Cost functions are determined considering Vehicle and personnel costs. The operation consists i a trip from the operative base to the collection area, returning to the disposal site (the operative base is on the way to the disposal site for any area) and returning to the collection area if necessary. Workers work in "normal hours" (the first 7 hours), 50% extra-hours (the two following hours) and 100% extra-hours (the following hours).

	Type of Hour		
	Normal	50%	100%
Chofer	3.38	5.07	7.61
Loader	2.87	4.30	6.45

Table 1 - Extra and normal hours costs.

Variable	Cost ($/km)
Amortization	0.58
Insurance	0.09
Licences	0.01
Conservation	0.13
Financial	0.01
Total fixed cost (Cfv)	0.82
Fuel	0.10
Lubricants	0.01
Filters	0.015
Lubrication	0.015
Tyres	0.02
Variable cost (Cvv)	0.16
Total	0.98

Table 2 - Vehicle costs.

Name.	Value	Ratio of Prod.	Description
Vor	35 km/h		Speed of vehicle out of route
Chof(n)	3.38 $		Cost of normal driver hour
Carg(n)	2.87 $		Cost of loader worker Normal hour
Chof(50)	5.07 $		Cost of driver 50%extra-hour
Carg(50)	4.3 $		Cost of loader 50% extra-hour
Chof(100)	7.61 $		Cost of driver 100%extra-hour
Carg(100)	6.45 $		Cost of loader 100% extra-hour
te2t	C/R+col. 2*2		Time for doing two trips to disposal site without taking load in the second
Cfv	0.36 $/km- for 9 ton vehicles 1.2 $/km- for 13 ton vehicles		Fixed costs of vehicle
Cvv	0.16 $/km		Variable costs of vehicles
Dpb	14 km		Distance from disposal site to operative base
Ts	1.1 hs		Time to go back and from disposal site to operative base (including waiting time at disposal site)

Table 3 - Nomenclature and formulas of the marginal costs table.

R indicates the considered productivity: the value of 1 means variable productivity: 2 ton/h for normal hours, 1.5 ton/hs for 50% extra hours and 1.3 ton/h for 100% extra hours. If it is 2, the productivity is constant = 2 ton/h.

(Variables are referred to as columns)

Variable	Name.	Value	Ratio of Prod.	Description
1	X			Distance from operative base to zone of recollection
2				Time of a trip to disposal site to recollection zone (back and from)
3		(9-te2t+Ts)*1.5	1.5-50% 1.3-100%	Tonnage to collect in 50% extra-hours with trucks of 9 tons
4		((Chn+2*Cn)*7+(Cfv+Cvv)*(X*2+Dpb*2))/9	1.5-50% 1.3-100%	Cost per ton for one trip (trucks of 9 tons)
5		((Ch100*Ts)+(Ch50+C50*2)*2+Cvv*(X*2+Dpb*2))/col 3	1.5-50% 1.3-100%	Cost per ton collected in 50% extra-hours and second trip(trucks of 9 tons)
6		((Ch50+2*C50)*2+(Ch100+2*C100)*(9-col 3)/1.3+Ts*Ch100+(Cvv)*(X*2+2*Dpb))/9	1.5-50% 1.3-100%	Cost per ton collected in 100% extra-hours and completing second load.
7		((Ch100*Ts)+(Ch50+C50*2)*2+Cvv*(X*2+Dpb*2))/col 6	2	Cost per ton collected in 50% extra-hours and second trip(trucks of 9 ton)
8		((Ch50+2*C50)*2+(Ch100+2*C100)*(9-col 6)/1.3+Ts*Ch100+(Cvv)*(X*2+2*Dpb))/9	2	Cost per ton collected in 100% extra-hours and completing second load.
9		((Chn+2*Cn)*7+(Cfv+Cvv)*(X*2+Dpb*2)+col10*C50+col11*Ch50)/13	2	Cost for a trip (trucks of 13 tons).
10		13/2+col 2-Ts-7		50% extra-hours of auxiliary workers in a truck of 13 tons (1 trip)
11		13/2+col 2-7		50% extra-hours of chofer in a truck of 13 tons (1 trip)

Table 4 - Variables for determination of marginal costs (Table 5).

X	Tzv(i)	Variable ratio of Prod.=Normal hours-2 ton/h 50% Extra-hours=1.5 t/h 100% extra-hour=1.3t/h				Constant Ratio of Productivity=2 ton/hour			13 Ton.-1 Trip		
		TONREC(50)	COS(1V)	COS(50)	COS(100)	TONREC(50)	COS(50)	COS(100)	COS(13)	HECAR(50)	HECHOF(50)
0	1,1	5,1	10,1	7,9	11,3	6,8	5,9	7,0	7,7		0,6
1	1,2	4,9	10,4	8,2	11,7	6,6	4,9	7,3	7,9		0,7
2	1,2	4,8	10,6	8,6	12,0	6,3	5,1	7,6	8,1		0,7
3	1,3	4,6	10,8	9,0	12,3	6,1	5,4	7,9	8,3		0,8
4	1,3	4,4	11,0	9,4	12,7	5,9	5,6	8,2	8,5		0,8
5	1,4	4,2	11,2	9,8	13,0	5,7	5,9	8,5	8,7		0,9
6	1,4	4,1	11,4	10,3	13,3	5,4	6,2	8,8	8,9		0,9
7	1,5	3,9	11,7	10,9	13,7	5,2	6,6	9,1	9,1		1,0
8	1,6	3,7	11,9	11,5	14,0	5,0	6,9	9,3	9,3		1,1
9	1,6	3,6	12,1	12,1	14,3	4,7	7,3	9,6	9,6		1,1
10	1,7	3,4	12,3	12,8	14,7	4,5	7,8	9,9	9,8	0,1	1,2
11	1,7	3,2	12,5	13,6	15,0	4,3	8,2	10,2	10,0	0,1	1,2
12	1,8	3,0	12,8	14,5	15,4	4,1	8,8	10,5	10,2	0,2	1,3
13	1,8	2,9	13,0	15,4	15,7	3,8	9,4	10,8	10,5	0,2	1,3
14	1,9	2,7	13,2	16,5	16,0	3,6	10,1	11,1	10,7	0,3	1,4
15	2,0	2,5	13,4	17,8	16,4	3,4	10,9	11,4	10,9	0,4	1,5
16	2,0	2,4	13,6	19,2	16,7	3,1	11,8	11,7	11,1	0,4	1,5
17	2,1	2,2	13,8	20,9	17,0	2,9	12,8	12,0	11,4	0,5	1,6
18	2,1	2,0	14,1	22,8	17,4	2,7	14,0	12,3	11,6	0,5	1,6
19	2,2	1,8	14,3	25,1	17,7	2,5	15,4	12,6	11,8	0,6	1,7
20	2,2	1,7	14,5	27,9	18,0	2,2	17,2	12,9	12,0	0,6	1,7
21	2,3	1,5	14,7	31,3	18,4	2,0	19,3	13,2	12,3	0,7	1,8
22	2,4	1,3	14,9	35,6	18,7	1,8	21,9	13,5	12,5	0,8	1,9
23	2,4	1,2	15,2	41,1	19,1	1,5	25,4	13,8	12,7	0,8	1,9
24	2,5	1,0	15,4	48,6	19,4	1,3	30,1	14,1	12,9	0,9	2,0
25	2,5	0,8	15,6	59,2	19,7	1,1	36,7	14,4	13,1	0,9	2,0

Table 5 - Costs per ton for different operative policies ($/ton).