Quinto Seminario de la Red Latinoamericana
“Optimización Discreta y Grafos: Teoría, Algoritmos y Aplicaciones”

8 y 9 de noviembre de 2012
Universidad Nacional de General Sarmiento – Buenos Aires, Argentina

Auspiciantes:

Universidad Nacional de General Sarmiento - Instituto de Ciencias

Instituto de Cálculo, FCEyN, UBA

Quinto Seminario de la Red Latinoamericana
"Optimización Discreta y Grafos: Teoría, Algoritmos y Aplicaciones"

Jueves 8 de noviembre de 2012 – Aula 3003

10:00 – 10:45	Min Chih Lin (Universidad de Buenos Aires, Argentina), "Dominating induced matchings"
10:45 – 11:30	Jayme Szwarcfiter (Universidad Federal de Río de Janeiro, Brasil), "On hull sets of graphs"
11:30 – 12:00	*Coffee break*
12:00 – 12:45	Rafael Epstein (Universidad de Chile, Chile), "Optimización de la logística de contenedores para una empresa naviera de gran tamaño"
12:45 – 14:30	*Almuerzo*
14:30 – 15:15	Martin Milanič (Universidad de Primorska, Eslovenia), "A hereditary view on efficient domination"
15:15 – 16:00	Marisa Gutiérrez (Universidad Nacional de La Plata, Argentina), "On rooted directed path graphs"
16:00 – 16:30	*Coffee break*
16:30 – 17:15	Boštjan Brešar (Universidad de Maribor, Eslovenia), "Median graphs and their cube graphs"
17:15 – 17:50	Nicolás Stier (Universidad Torcuato Di Tella, Argentina y Universidad de Columbia, EEUU), "The competitive facility location problem in a duopoly"

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Viernes 9 de noviembre de 2012 – Aula 3001

10:00 – 10:45	Flavia Bonomo (Universidad de Buenos Aires, Argentina), "Large minimum clique cover in claw-free perfect graphs via 2-SAT"
10:45 – 11:30	Valeria Leoni (Universidad Nacional de Rosario, Argentina), "Variaciones de los problemas de dominación y empaquetamientos en grafos"
11:30 – 12:00	*Coffee break*
12:00 – 12:45	Andrés Weintraub (Universidad de Chile, Chile), "Cadenas de suministro en recursos naturales"
12:45 – 14:30	*Almuerzo*
14:30 – 15:15	Marcelo Mydlarz (Universidad Nacional de General Sarmiento, Argentina), "Una extensión al problema del 3D-matching"
15:15 – 16:00	Mario Valencia-Pabon (Universidad Paris-Nord, Francia), "On some coloring problems on Hypercubes"
16:00 – 16:30	*Coffee break*
16:30 – 17:15	Guillermo Durán (Universidad de Buenos Aires, Argentina y Universidad de Chile, Chile), "Optimizing salmon farm cage net management using integer programming"

Min Chih Lin (Universidad de Buenos Aires, Argentina)

Dominating induced matchings

Let G be a simple weighted undirected graph, i.e., a graph without loops and multiple edges with vertex set V, edge set E and a weight function $w : E(G) \to \mathbb{R}$. Given an edge $e \in E$, we say that e dominates itself and every edge sharing a vertex with e. An induced matching in G is a subset of edges such that each edge of G is dominated by at most one edge of the subset. The problem of determining whether a graph has a dominating induced matching, i.e., an induced matching that dominates every edge of the graph is also known in the literature as *dominating induced matching* (DIM for short) or *efficient edge domination*. This problem is NP-Complete. In this work we study the weighted version of DIM, this is, find a DIM M such that the sum of weights of its edges is minimum between all DIM's if any and we proposed two exact algorithms to determine a minimum DIM. One of them with time complexity $O(1.44225^n m)$ and it can be extended to count the number of DIMs of G within this same complexity. Also, we give a tight upper bound for the number of DIMs.

Joint work with Michel Mizrahi, Verónica Moyano, and Jayme Szwarcfiter.

Jayme Szwarcfiter (Universidad Federal de Río de Janeiro, Brasil)

On hull sets of graphs

A graph convexity $(G, \mathcal{C})$ is a graph G together with a collection of subsets $C \subseteq \mathcal{C}$, called *convex sets*, such that $\emptyset, V(G) \in \mathcal{C}$ and $\mathcal{C}$ is closed under intersections. For an arbitrary set $S \subseteq V(G)$ the *hull of* S is the smallest convex set containing S. In particular, when the hull of S equals $V(G)$ then S is a *hull set* of G. The cardinality of the least hull set of G is the *hull number* of the graph. In this talk, we consider complexity aspects related to the determination of the hull number of a graph, under some different graph convexities, such as the geodetic convexity, monophonic and P3 convexities. We describe NP-hardness results, polynomial-time solvable cases and bounds. In addition, we also describe some additional convexity parameters, as the geodetic and convexity numbers of a graph.

Joint work with Mitre C Dourado and Dieter Rautenbach.

Rafael Epstein (Universidad de Chile, Chile)

Optimización de la logística de contenedores para una empresa naviera de gran tamaño

Mostraremos el problema logístico que enfrenta una naviera para determinar los niveles de inventario de contenedores vacíos en los diferentes puertos donde ofrece servicio y la mejor política de reposicionamiento de contenedores vacíos. Este problema es difícil porque combina problemas analíticos como la predicción de la demanda de contenedores y la optimización de los flujos de los contenedores considerando múltiples períodos, orígenes y destinos. Adicionalmente, nos enfrentamos a una operación de carácter global que efectivamente es 24 por 7, donde tenemos que coordinar diferentes tomadores de decisiones, localizados en todo el mundo, con realidades y culturas también muy distintas.

Martin Milanič (Universidad de Primorska, Eslovenia)

A hereditary view on efficient domination

An efficient dominating set (or perfect code) in a graph is a set of vertices the closed neighborhoods of which partition the graph's vertex set. It is NP-complete to determine whether a given graph contains an efficient dominating set. We study the class of hereditary efficiently dominatable graphs, that is, graphs every induced subgraph of which contains an efficient dominating set. Based on a decomposition theorem for (bull, fork, C_4)-free graphs, we derive the forbidden induced subgraph characterization of hereditary efficiently dominatable graphs. We also present a polynomial time algorithm for finding an efficient dominating set (if one exists) in a class of graphs properly containing the class of hereditary efficiently dominatable graphs, by reducing the problem to the maximum weight independent set problem in claw-free graphs.

Marisa Gutiérrez (Universidad Nacional de La Plata, Argentina)

On rooted directed path graphs

A graph is *chordal* if it contains no cycle of length at least four as an induced subgraph. A classical result states that a graph is chordal if and only if it is the (vertex) intersection graph of a family of subtrees of a tree. It is easy to see that any *interval graph* is a chordal graph considering that it is the intersection graph of a family of subpaths of a path.

An *asteroidal triple* in a graph G is a set of three non-adjacent vertices such that for any two of them there exists a path between them that does not intersect the neighborhood of the third.

Lekkerkerker and Boland proved that a chordal graph is an interval graph if and only if it contains no asteroidal triple. As byproduct, they found a characterization of interval graphs by forbidden induced subgraphs.

Another natural subclass of chordal graphs are path graphs. A graph is a *path graph* if it is the intersection graph of a family of subpaths of a tree. Lévêque, Maffray and Preissman, found the characterization of path graphs by forbidden induced subgraphs but there is still no nice characterization in terms of forbidden asteroids for this class.

Two subclasses of path graphs have been defined when the host tree is a directed graph. Firstly, a graph is a *directed path graph* if it is the intersection graph of a family of directed subpaths of a directed tree. Panda found the characterization of directed path graphs by forbidden induced subgraphs and then Cameron, Hoáng and Lévêque gave a characterization of this class in terms of forbidden asteroidal triples. Secondly, a graph is a *rooted path graph* if it is the intersection graph of a family of directed subpaths of a rooted tree. By definition we have the following inclusions between the different considered classes (and these inclusions are strict):

$$\text{interval} \subset \text{rooted path} \subset \text{directed path} \subset \text{path} \subset \text{chordal}$$

Characterizing rooted path graphs by forbidden induced subgraphs or forbidden asteroids are open problems. It is certainly too difficult to characterize rooted path graphs by forbidden induced subgraphs as there are too many (families of) graphs to exclude but Cameron, Hoáng and Lévêque gave a conjecture which proposes a characterization of these graphs in terms of forbidden asteroids. This conjecture is in fact an attempt to characterize directed path graphs that are non rooted path graphs by forbidding particular type of asteroidal quadruples. An *asteroidal quadruple* in a graph G is a set of four non-adjacent vertices such that any three of them is an asteroidal triple. In its original form, this conjecture is incomplete as was shown in. Gutierrez, Lévêque and Tondato proved also that every directed path graph that is not a rooted path graph has an asteroidal quadruple. As suggested by the conjecture, a characterization by forbidding particular type of asteroidal quadruples may hold.

For this purpose, we are studying models of directed path graph non-rooted directed path graph. We prove that such models will have at least four leaves and the graph must have an asteroidal quadruple. In case that the leafage of the graph is four, we proved that it has an unique asteroidal quadruple. On the other hand, some paths in the graph which force the direction in any DV-model will be relevant. Clearly, one of these is a path of three vertices between two non-adjacent vertices. However, as was proved by Cameron, Hoáng and Lévêque, there are other types of paths with this property. Finally, we propose the following conjecture: any DV non RDV graph has an asteroidal quadruple with two pairs of forced paths, proving it for graphs with leafage four.

In cooperation with Silvia Tondato and Benjamin Lévêque.

Boštjan Brešar (Universidad de Maribor, Eslovenia)

Median graphs and their cube graphs

Abstract Median graphs are an important class of graphs that appear in different guises and applications, and relate to many other mathematical structures. One of the earliest characterizations of median graphs due to Isbell states that hypercubes form basic building blocks from which any (finite) median graph can be constructed by successive use of convex amalgamations. In this talk we give a brief survey on median graphs, and concentrate on the role of their maximal hypercubes. We focus on the so-called cube graphs, i.e., intersection graphs of maximal hypercubes, of median graphs, which yield many interesting connections with classes of Helly graphs, dually chordal graphs, diamond-free graphs, clique-graphs etc. Open problems and results on related classes of graphs are presented along the way.

Nicolás Stier (Universidad Torcuato Di Tella, Argentina y Universidad de Columbia, EEUU)

The competitive facility location problem in a duopoly

We consider a facility location game on a network in which consumers who are located on vertices wish to connect to the nearest facility. Knowing this, each competitor must choose a vertex to locate its facility, hoping to capture the largest-possible market share. The game resembles the classic Hotelling model of spatial competition. Focusing in a duopoly, we study the existence and the spatial location of pure-strategy Nash equilibria, for progressively more complicated classes of networks.

The case of trees is well-studied: equilibria are characterized by medians –nodes that minimize the distance to consumers– and hence always exist. For cycles, we find that an equilibrium exists when there is at least one vertex with a sufficiently-big demand, in which case it must also be a median. For more general graphs, we construct a tree of maximal bi-connected components and apply the results for trees and cycles to get sufficient conditions for equilibrium existence. This provides a complete and efficient characterization of equilibria for cactus graphs and other generalizations. As before, at equilibrium both competitors locate their facilities in medians.

The previous results generalize the classic study of Hotelling from a line to more complicated networks. Exploring this further, we establish a connection for additional classes of graphs such as quasi-median graphs, median graphs, Helly graphs, and strongly-chordal graphs, which include lattices and other naturally-occurring topologies in real-world networks. Furthermore, we establish a stronger result for strongly-chordal graph: as for trees, any median leads to an equilibrium.

Finally, we show that removing edges from a cactus increases consumer cost. This precludes situations like the Braess paradox, whereby removing an edge can improve the situation. This result implies that trees are worst-case networks because they are minimal instances with respect to inclusion. While equilibria can be arbitrarily inefficient with respect to centralized solutions, we quantify the resulting worst-case inefficiency with parametric upper bounds that depend on topological parameters of the network.

Flavia Bonomo (Universidad de Buenos Aires, Argentina)

Large minimum clique cover in claw-free perfect graphs via 2-SAT

For a perfect graph G, and given a weight function w on the vertices of G, linear programming duality ensures that the weight of a minimum weighted clique cover (an assignment of a non-negative weight y_C to each clique C of G, such that, each vertex v is covered by at least $w(v)$ cliques and that minimizes $\sum_C y_C$) is the same as the weight of a maximum weighted stable set (a set of pairwise nonadjacent vertices S that maximizes $\sum_{v \in S} w(v)$). Moreover Grötschel, Lovász and Schrijver gave in 1988 a (non combinatorial) algorithm, building upon Lovász's theta function, to compute both solutions. It is a major open problem in combinatorial optimization whether there exist polynomial time combinatorial algorithms for those two problems.

For particular classes of perfect graphs, such algorithms exist. This is the case, for instance, for claw-free perfect graphs (a graph is claw-free if none of its vertices has a stable set of size three in its neighborhood). Indeed, there are several combinatorial algorithms for solving the maximum weighted stable set problem (the fastest algorithm due to Faenza, Oriolo and Stauffer in 2011 runs in time $O(|V(G)|^3)$, while, to the best of our knowledge, the only combinatorial algorithm for the minimum weighted clique cover is due to Hsu and Nemhauser in 1984 and runs in time $O(|V(G)|^5)$. This latter algorithm is based on a clever use of complementary slackness in linear programming, combined with the resolution of several maximum weight stable set problems.

In this work, we propose a new approach to the minimum weighted clique cover problem in claw-free perfect graphs. We basically reduce the problem to the question whether a given maximal stable set S, is of maximum weight. In the cardinality case, we answer this question by solving a suitable 2-SAT instance. In particular, we show that a truth assignment to the 2-SAT instance can be directly translated into a minimum clique cover of size $|S|$, while an unsatisfiable instance "returns" a path that is augmenting with respect to S. This yields a new combinatorial algorithm that produces both a minimum cover and an maximum stable set of a claw-free perfect graph G in time $O(|V(G)|^3)$, in the spirit of the augmenting path algorithm for maximum bipartite matching and minimum vertex cover. In the weighted case, the question reduces to a kind of "weighted" 2-SAT problem: given a polyhedra of the form $Ax \leq b$ with A a $p \times q$ {0,1,-1} matrix, with at most two non-zero element per row, and b integer, find an integer solutions to the system, if any. This latter problem was first solved by Schrijver in 1991 (and also Peis in 2007), and then independently considered by several people in the Constraint Programming Community, like for instance Schutt and Stuckey in 2010. We propose here a novel approach that takes advantage of the structure of the polyhedron when dealing with the minimum weighted clique cover problem. It follows that a minimum weighted clique cover of a claw-free perfect graph G can be found in time $O(|V(G)|^3)$.

Joint work with Gianpaolo Oriolo, Claudia Snels and Gautier Stauffer.

Valeria Leoni (Universidad Nacional de Rosario, Argentina)

Variaciones de los problemas de dominación y empaquetamientos en grafos

El estudio de la dominación en grafos, comenzado por Claude Berge en 1958, surgió en parte como resultado del estudio de juegos y matemática recreativa. En la actualidad, sus numerosas aplicaciones han llevado a definir diversas variaciones del mismo.

En esta charla se muestra un estudio sobre algunas de estas variaciones, las cuales tienen que ver con modelizaciones de problemas de ubicación de servicios en lugares que cuentan con una capacidad limitada en su “vecindad”. Entre otros, ubicación de contenedores de basura en las esquinas de un barrio o de sensores en una instalación.

También presentamos problemas “simétricos” en su definición a los anteriores, introducimos el estudio de sus complejidades computacionales y mostramos su relación con los problemas de dominación.

Por último, mostramos algunas generalizaciones naturales de estos problemas. Nos interesa avanzar sobre el estudio de sus complejidades computacionales y presentar algunos problemas abiertos.

Andrés Weintraub (Universidad de Chile, Chile)

Cadenas de suministro en recursos naturales

Se analiza el funcionamiento de la cadena de suministro en manejo forestal, minero y cultivo de salmones, que en casi todos los casos no se ha integrado. Se presenta el manejo actual de cada cadena, mostrando las deficiencias al manejar cada componente en forma separada. Se muestran los modelos desarrollados en cada caso para coordinar la cadena, incluyendo las dificultades algorítmicas que presentan. Se compara la propuesta de modelos en que se integra la cadena, con el caso no integrado. En particular, un caso de minería en el que sí se implementó la integración de la extracción de cobre con el proceso aguas abajo. Se discute cómo se pueden considerar aspectos de incertidumbre en cada sector, en el caso forestal y minero incertidumbre de mercado (precios) y en el caso de salmones incertidumbre por perecibilidad a causa de un virus.

Estos desarrollos se han realizado con múltiples colegas y tesistas.

Marcelo Mydlarz (Universidad Nacional de General Sarmiento, Argentina)

Una extensión al problema del 3D-matching

Consideremos el siguiente caso particular del problema de 3D-matching: dados un conjunto T de triplas ordenadas y un entero t, decidir si existe un subconjunto M de T tal que $|M| = t$ y ningún par de triplas de M coincidan en dos coordenadas. Veremos una aplicación, algunas propiedades estructurales del problema, y deduciremos que para $k \geq 3$ y $p < k$ el problema correspondiente con k-tuplas y p coordenadas es $\mathcal{NP}$-completo.

Mario Valencia-Pabon (Universidad Paris-Nord, Francia)

On some coloring problems on Hypercubes

An n-dimensional Hypercube is a graph whose vertices are the vectors of the n-dimensional vector space over the field $GF(2)$. There is an edge between two vertices of the n-dimensional Hypercube whenever their Hamming distance is exactly 1, where the Hamming distance between two vectors is the number of coordinates in which they differ. In this talk, I will discuss some open coloring problems on Hypercubes by using algebraic techniques. In particular, some bounds for the chromatic number of powers of Hypercubes and for the packing chromatic number of this family of graphs will be presented.

Guillermo Durán (Universidad de Buenos Aires, Argentina y Universidad de Chile, Chile)

Optimizing salmon farm cage net management using integer programming

Salmon farming in Chile constitutes one of the nation's principal food exporting sectors. In the seawater stage, one of the most important in the farm production chain, salmon are cultivated in floating cages fitted with nets that hold the fish during the entire grow-out process. The maintenance of the cage nets is carried out at land-based facilities. This article reports on the creation of an integer programming tool for grow-out centers that optimizes resource use, improves planning and generates economic evaluations for supporting analysis and decision-making relating to the maintenance, repair and periodic changing of cage nets. The tool prototype was tested in a single operating area of one of Chile's largest salmon farmers. The results demonstrated a reduction in net maintenance costs of almost 18%, plus a series of important qualitative benefits. Implementation of the tool by farm operators awaits the end of the current crisis in the industry.

Joint work with F. Cisternas, D. Delle Donne, C. Polgatiz, and A. Weintraub